

# OPTIMUM RUNNING OF HEAT PUMP UNDER LOW-LOAD CONDITION

*Li Bing-xi, Wang Yong-biao, Yuan Di*

*School of Energy Science and Engineering, Harbin Institute of Technology,  
92 West Dazhi Street, Harbin 150001, People's Republic of China*

## ABSTRACT

A running model of heat pump that operates at the low load status (operating load is not more than half of system's designing capacity) is presented in this paper. Full load intermittent running model will result in that the system starts and stops frequently and consumes a large amount of extra energy, thus partial load intermittent running model will be introduced in this paper. The partial operating load of the system at the low load status is calculated based on optimization theory, in which energy consumed function of the system and operating load are regarded as the object function and optimized parameter, respectively. The result shows that there is an optimum running load making the system running in the lowest energy consuming and can greatly improve the system's performance in the case of a small heat capacity and a great heat output.

**Key Words:** heat pump, low load, optimum running, optimum value.

## 1 FOREWORD

Ground-source heat pump (GSHP) is a kind of new style environmental protecting air-conditions that utilize geothermal energy resources to provide heating and cooling for buildings. Because their heat sources are stable, invariable and of a higher temperature than those of other heat pumps, the coefficient of performance (COP) values of GSHP systems are much higher than those of the heat pumps with other heat sources no matter in heating mode or cooling mode. (Zhang 1999; Swardt 2001; Shou and Chen 2001) Thus, GSHP systems are attractive in cold climate region, and they are the optimal substitutes for mini-boilers to heat buildings. In general, air-conditioning design load in summer is calculated according to the load in the hottest month, and similarly, heating design load in winter is calculated according to the load in the coldest month. In north areas of cold climate, air-conditioning design load and heating design load are approximately the same, however, the cold load is much less than the design load during a majority of running period in summer, and what's more, there is distinct difference of temperature between day and night, which makes loads unequal, and a heating period for half a year, all those make a GSHP runs at a low load status in the beginning and at the end of the heating period. Running in the low load will lead to a decreasing efficiency of total system. It is viable to adopt a parallel system to provide heat to an expansive place (Wang et al. 1997), in which partial running for partial load, however, for individual heating, where single machine is common used, system load is regulated by continuous changes of the amount of input gases when compressor load in more than half of the design one; otherwise, because of the less than 50% amount of input gases and the poor system performance when compressor load is less than 50% of the design load, we can not satisfy the load's demand by regulating the amount of input gases of the compressor and usually adopt the full load intermittent running method. Considerable of energy is consumed to engender a differential pressure on both sides of the throttle during its start-up process; heat (or cold) output is increasing along with the time's passing until the system reaches the full load under a stable running status. To improve the disadvantage of frequently start-up and high energy costs in full load intermittent running model, this paper, based on optimization theory, presents an appropriate running load in low load running mode, which is effective to minimize the energy

costs of the system.

## 2 OBJECT FUNCTION AND RESTRICTION CONDITIONS

### 2.1 Establishment of Object Function

a. Efficiency function of electromotor:

$$\eta_{mo} = f(P_{el} / P_{eln}) \quad (1)$$

Where  $P_{el}$  is the running power of electromotor,  $P_{el} = P / COP$ ;  $P_{eln}$  is the nominal electric power,  $P_{eln} = P_r / COP_r$ ;  $P$  is running load of system,  $P_r$  is rated load of system,  $COP$  is defined as the ratio of heat output to axis work, and  $g(x) = COP / COP_r$ ; So,

$$f(P_{el} / P_{eln}) = f\left(\frac{P}{COP} / \frac{P_r}{COP_r}\right) = f\left(\frac{P}{COP} \cdot \frac{1}{\frac{P_r}{COP_r}}\right) \quad (2)$$

In this paper, water flux in condenser is assumed to be constant, and piping loss is eliminated from consideration.

$$Q = k \cdot A \cdot \Delta T \quad (3)$$

$$\Delta T = (T_s + T_{re}) / 2 - T_i \quad (4)$$

Where  $k$  is thermal conductivity, and  $A$  is heat exchange area; with equations above, it is observed that the running load of system is in direct proportion to  $\Delta T$ . Supply water and return water temperatures as well as condensation temperature of system drop with system running load's decreasing. Because soil is of a relatively stable temperature in GSHP systems, it is acceptable to assume a constant evaporation temperature. From the following equation:

$$COP = \frac{1}{1 - T_e / T_c} \quad (5)$$

We know that system's ideal  $COP$  value is increasing by the decreasing of  $T_c$ , and meanwhile refrigerant flux drops with the load's decreasing. In this paper, the influence to system  $COP$  value, (caused by the change of refrigerant flux), is neglected when compressor inspiration is more than 50%. Besides, it is assumed that real heating coefficient is in direct proportion to ideal heating coefficient. So,  $g(x)$  is expressed as:

$$g(x) = \frac{COP}{COP_{eln}} = \frac{1}{1 - T_e / T_c} \bigg/ \frac{1}{1 - T_e / T_{c,eln}} \quad (6)$$

Suppose the difference in condensation temperature and hot water temperature is constant,

$$T_c - (T_{sy} + T_{re}) / 2 = \Delta T_c \quad (7)$$

and  $T_c$  is computed from equation (3), equation (4) and equation (7):

$$T_c = \frac{P}{k \cdot A} + T_i + \Delta T_c \quad (8)$$

and function  $g(x)$  of  $P$  can be determined by equation (6) and equation(8).

b. Energy costs ( $E$ )

$$E = P_{el} / f(P_{el} / P_{eln}) = \frac{P}{COP_r \cdot f(\frac{P}{P_r} \cdot \frac{I}{g(x)}) \cdot g(x)} \quad (9)$$

c. Object function

$$J = \int \frac{P}{COP_r \cdot f(\frac{P}{P_r} \cdot \frac{I}{g(x)}) \cdot g(x)} dt \quad (10)$$

## 2.2 Restriction Condition

a. Indoor heat balance

$$\rho c V \frac{dT}{d\tau} = P \cdot r(\tau) - Q_o \quad (11)$$

Where  $\rho c V$  is indoor thermal capacity,  $\text{kJ}/^\circ\text{C}$ . Heat is transferred from heat exchangers to objects indoors through air, and thermal conductivity and temperature difference values are relatively small between airs and indoors, so a effective coefficient that is less than 1 is needed to be multiplied by when thermal capacity is taken into account, and indoor thermal capacity is the sum of thermal capacities of air and other objects.  $r(\tau)$  is the relationship of the ratio of heat output at start-up state to that in the normal running period versus time,  $r(\tau)=1$  when system is in the normal running status;  $T$  is indoor air temperature, and  $\tau$  is the time.

b. Indoor temperature precision requirement

$$|T - 20| < \Delta T \quad (12)$$

Where  $\Delta T$  is permissible value of fluctuation of temperature,  $^\circ\text{C}$ ; indoor design temperature is  $20^\circ\text{C}$ .

c. Safety running requirement of system

It is required that system should be started up only after it is laid off for a period of time, and  $Time$  is the interval:

$$Time > \beta \quad (13)$$

Where  $\beta$  is permissible minimum interval.

### 3 SIMULATED CALCULATION AND ANALYSIS

In this paper, heating load in winter is set to be 7.0KW

#### 3.1 Start-up Energy Function and Electromotor Efficiency

We suppose that startup period lasts 4 minutes (Chen et al. 1997); start-up function 1 and 2 are linear function and parabolic function respectively (Li et al. 2001), which are shown in figure. 1. The electromotor efficiencies are plotted against the corresponding power ratio,  $P_{el}/P_{eln}$  (Miao and Wu 2001). By using the method of least square, curves in figure. 2 are fitted into unitary cubic equations in this paper.

Curve 1:

$$Y=0.6406X^3-1.644X^2+1.455X+0.3409 \quad (14)$$

Curve 2:

$$Y=0.3569X^3-1.0427X^2+1.145X+0.3160 \quad (15)$$

Where  $X=P_{el}/P_{eln}$ ,  $Y=\eta_{mo}$ .

#### 3.2 Running Energy Costs

Under the condition of a constant environmental temperature and a constant indoor thermal capacity, for low load intermittent running GSHP, the changing course of indoor air temperature is circulatory.

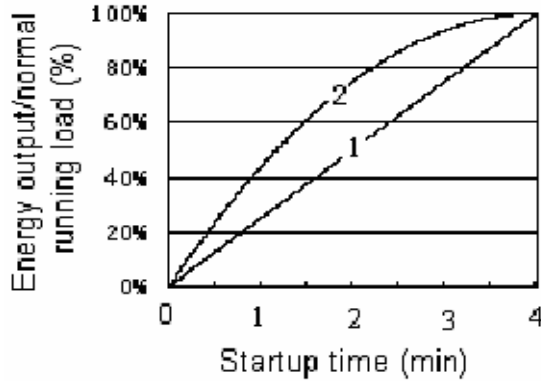


Fig. 1. Energy output function of GSHP in startup electromotor

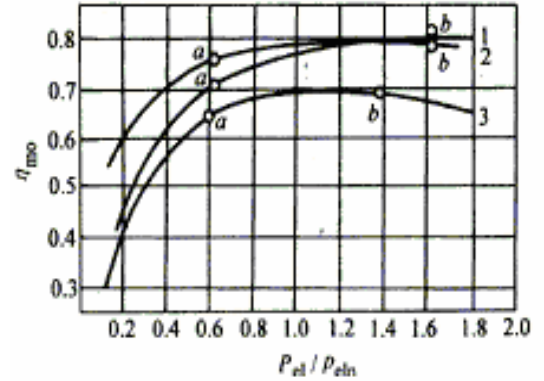


Fig. 2. Efficiency function of GSHP

Initial temperature is assumed to be  $20+\Delta T$ . As for heat output during the start-up process, it needs a period of time to increase from 0 to the running load, thus it is necessary to start up GSHP in advance before indoor air temperature drops lowermost in order to meet the indoor temperature precision requirement, and pre-startup time is relevant to start-up function, running load and heat output.

Since GSHP system begins to start, Temperature indoors keeps on decreasing until  $P \cdot r(t) = Q_o$ . When  $P \cdot r(t) > Q_{output}$ , indoor temperature turns to increase because  $\rho c v (T_n - T_{n-1}) = P \cdot r(\tau) - Q_o$ . So,

$P \cdot r(t) = Q_o$  is a turning point. Time  $t_{start}$  at the turning point can be evaluated from equation:  $r(t) = \frac{Q_o}{P}$ , and indoor temperature at the turning point is set to be the minimum value:  $20-\Delta T$ . In order to ensure

that indoor temperature at  $t_{start}$  is not lower than permissible minimum temperature, indoor temperature  $T_{start}$  in system's start-up period can be computed from:

$$\rho cv [T_{start} - (20 + \nabla T)] = - \int_0^{t_{start}} (P \cdot r(\tau) - Q_o) d\tau \quad (16)$$

GSHP stops when indoor temperature varies from  $20 + \Delta T$  to  $T_{start}$ , there is still a heat output, however. So, the time that indoor temperature varies from  $20 + \Delta T$  to  $T_{start}$ , i.e. the stop time  $t_{stop}$  in a circle, can be calculated from indoor heat balance equation (11). In system's startup process, i.e., from  $t_{start}$  to stable heat output point  $t_{stable}$ , the elevated value of indoor air temperature is:

$$\Delta T_{air1} = \frac{\int_{t_{start}}^{t_{stable}} (P \cdot r(\tau) - Q_o) d\tau}{\rho cv} \quad (17)$$

If  $\Delta T_{air1} < 2 \cdot \Delta T$ , it indicates that indoor air temperature has not been up to the permissible maximum value ( $20 + \Delta T$ ) during the period from start-up to normal running, it is necessary, in another word, that GSHP must keep on running. The elevated temperature value during the period of stable running until its stop is  $\Delta T_{air2}$ :

$$\Delta T_{air1} + \Delta T_{air2} = 2 \cdot \nabla T \quad (18)$$

With equation (4-10), it is observed that:

$$t_{stable} = \frac{\rho cv (2 \cdot \Delta T - \Delta T_{air1})}{P - Q_o} \quad (19)$$

So the total running time is  $4 + t_{stable}$  minutes, ( $t_{start}=4$  minutes), stop time is  $t_{stop}$  minutes and the total circle time is  $4 + t_{stable} + t_{stop}$  minutes.

Thus, energy costs in one hour are ( $t_{start}=4$  minutes):

$$E = \frac{60}{4 + t_{stable} + t_{stop}} \cdot \frac{P}{COP_{rated} \cdot f\left(\frac{P}{P_r} \cdot \frac{1}{g(x)}\right) \cdot g(x)} \cdot (4 + t_{stable}) \quad (20)$$

When  $Q_o = 2$  kW,  $\Delta T = 2$  °C and indoor thermal capacity  $\rho cV$  is 500 kJ/°C, 600 kJ/°C, 700 kJ/°C, 800 kJ/°C respectively, energy costs curves for start-up function 1 and electromotor curve 1 are plotted in figure. 3. It is observed that every heat capacity has a minimum energy costs value, which indicates that there is an optimized running load (optimum value) aiming at every heat load to minimize the running energy costs of system. Optimized running loads of GSHP are not exactly the same for different heat capacities. Moreover, optimized running load increases with the increasing of heat capacity, and system energy costs are decreasing gradually with the increasing of heat capacity.

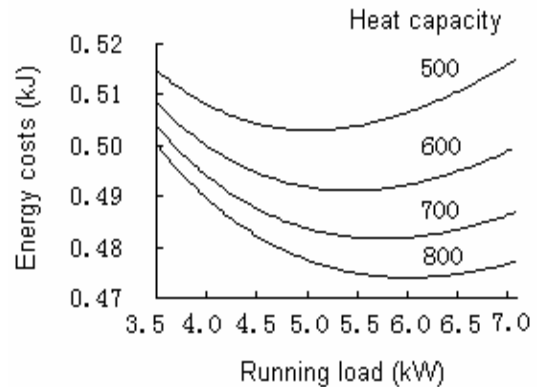


Fig. 3. Energy costs for different running load

### 3.3 Optimum Value for System Running

As long as running loads satisfy equation (21) and equation (22), they are the optimum values under that condition.

$$\frac{\partial J}{\partial P} = 0 \quad (21)$$

$$\frac{\partial^2 J}{\partial P^2} > 0 \quad (22)$$

Under low load condition, the optimized running load value chart for start-up function 1, electromotor efficiency function 1 and  $\Delta T = 2.0^\circ\text{C}$  is shown in figure. 4-1, where X axis represents indoor heat capacity,  $\text{kJ}/^\circ\text{C}$ , Y axis represents heat load,  $\text{kW}$ , and Z axis is the optimized running load of system,  $\text{kW}$ . Optimum value is gradually increasing as indoor heat capacity increases under the condition of a constant heat load in the direction of Y-axis. It can be observed from figure. 4-1 that, in the case of a constant indoor heat capacity, the optimized running load is kept constant in the beginning, and then it decreases gradually as heat load increases; simultaneously, the curvatures of optimized running load curves for different heat loads are decreasing as indoor heat capacity increases.

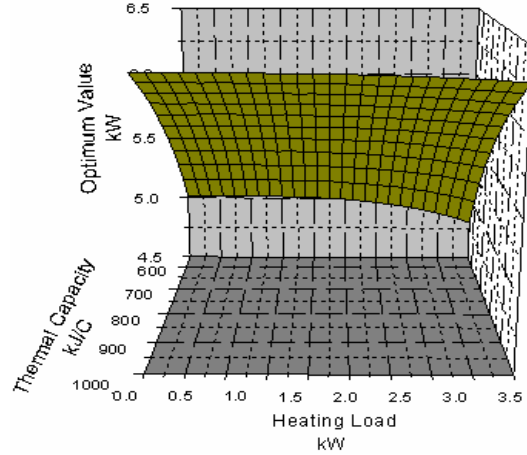


Fig. 4-1. Optimum value chart

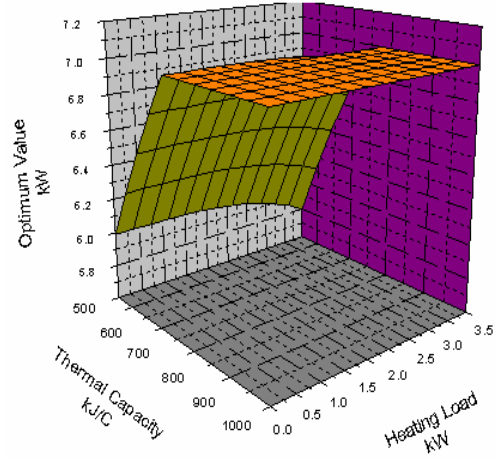


Fig. 4-2. Optimum value region chart of system (start-up function 1, electromotor curve 2 and  $\Delta T = 2.0^\circ\text{C}$ )

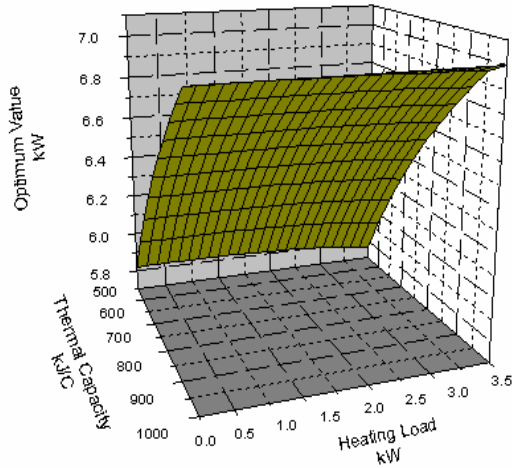


Fig. 4-3. Optimum value region chart of system

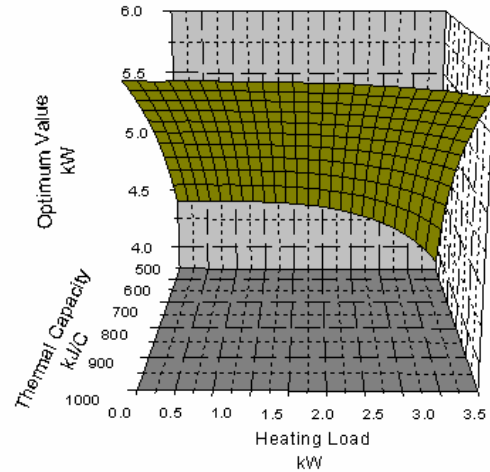


Fig. 4-4. Optimum value region chart of

(start-up function 2, electromotor curve 2  
and  $\Delta T = 2.0^{\circ}\text{C}$ )

(start-up function 1, electromotor curve  
1 and  $\Delta T = 1.5^{\circ}\text{C}$ )

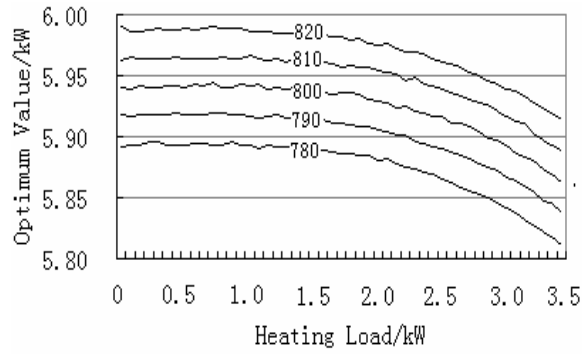
The optimized running load charts for the case of start-up function 1 and electromotor efficiency function 2 and the case of start-up function 2 and electromotor efficiency function 1 under the condition of  $\Delta T = 2.0^{\circ}\text{C}$  are plotted in figure.4-2 and figure.4-3, respectively. Through comparing figure.4-1 with figure.4-2, we know that, when start-up function 1 is kept constant and electromotor efficiency curve changes from curve 1 to curve 2, curved surface of the optimized running load of GSHP shifts upwards, and system optimized running load attains to the rated load, which means that system, in the scopes of large heat capacity, can operates under full load status with a minimum energy costs. Similarly, through comparing figure.4-1 with figure.4-3, we know that, when electromotor efficiency curve 1 is kept constant and start-up function changes from function 1 to function 2, curved surface of the optimized running load of GSHP shifts upwards, which means that system optimized running load has increased; a comparison of these two results reveals that the change of electromotor efficiency influences more on the optimized running value than the change of start-up function does.

The optimized running load chart (under the condition of low load and for the case of start-up function 1, electromotor efficiency function 1 and  $\Delta T = 1.5^{\circ}\text{C}$ ) is plotted in figure.4-4. The comparison of figure.4-1 to figure.4-4 shows that, the optimized running loads are different with the difference of  $\Delta T$  even for the same start-up function and electromotor efficiency function, moreover, the optimized running load of GSHP is decreasing gradually with the decrease of  $\Delta T$  value.

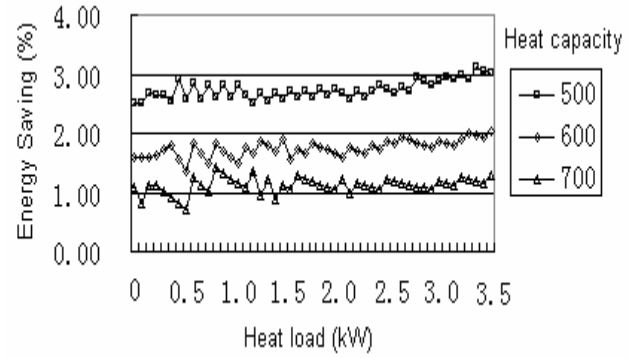
### 3.4 Results and Discussion

It is observed from the foregoing analysis that, for the case of a constant heat capacity, the change value of the optimized load of GSHP is insignificant even as heat load rises from 0 to 3.5 kW. Even a slightly change in heat capacity cannot influence the optimized running load much. Figure.5 shows the optimum value for the case of start-up function 1, electromotor curve 1 and  $\Delta T = 2.0$  when heat capacity fluctuates around  $800 \text{ kJ}/^{\circ}\text{C}$ . It shows that the change value of the optimized running load will not exceed  $0.05 \text{ kW}$  with an indoor heat capacity increase or decrease of  $10 \text{ kJ}/^{\circ}\text{C}$ . Simultaneously, for the same heat capacity, the discrepancy between the maximum and minimum optimized running load will not exceed  $0.1 \text{ kW}$ , and it increases with the decrease of heat capacity and decreases with the increase of heat capacity, of course. Obviously, for GSHP, heating object is fixed, and indoor heat capacity can be generally kept in a slightly fluctuant scopes. In practical projects, how to control conveniently is taken into account, and the same optimized running load is constant for the same heating object system, all those account for a simplified control and energy saving system, consequently.

Figure 6 reveals a relative percentage for the decrease of energy costs when running under the condition of optimum value to that running under maximum load. It is observed that the less heat capacity is, the better energy saving effects are; and for the same heating object, the higher heat load is, the better trend of energy saving effects is, as well.



**Fig. 5. Optimized running load of GSHP**



**Fig. 6. Energy saving status for optimized system running**

## 4 CONCLUSIONS

1. For each heat load value, there is an optimized running load (optimum value) to make GSHP operate with the minimum energy costs;
2. For the same heating object, its indoor heat capacity can fluctuate slightly around a special value, thus the optimized running load can be regarded as approximate invariable when heat load changes from 0 to 50%, and that the optimized running of GSHP will not be disturbed;
3. The electromotor efficiency functions of system affect the system optimum value and its sub areas to a great extent. The efficiency's drop will account for the rise of the optimized running load value; system start-up energy output function curves and demand temperature precision have a relatively little effect on the optimized running load value, the changes of these two factors can only shift the optimum value curved surface up and down;
4. Optimized running mode can greatly improve the system's performance in the case of a little heat capacity and a great heat output.

## NOMENCLATURE

### Symbols

$\beta$	=Permissible minimum interval
$\eta_{mo}$	=Electromotor efficiency
$\rho cv$	=Thermal capacity, kJ/°C
$\tau$	=Time
$A$	=Heat exchange area
$COP$	= Coefficient of performance, i.e. ratio o heat output to axis work
$E$	=Energy costs
$J$	=Object function
$k$	=Thermal conductivity
$P$	=Running load of system
$Q$	=Quantity of heat, kJ
$T$	=Temperature, °C

### Subscripts

$c$	=Condensation
$e$	=Evaporation



<i>el</i>	=Electromotor
<i>i</i>	=Indoor
<i>n</i>	=Nominal
<i>o</i>	=output
<i>r</i>	=Rated
<i>re</i>	=return
<i>s</i>	=supply
<i>stable</i>	=stable running period
<i>start</i>	=start time
<i>stop</i>	=stop time

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