# FULLY ANALYTICAL FINITE LINE SOURCE SOLUTION FOR FAST CALCULATION OF TEMPERATURE RESPONSE FACTORS IN GEOTHERMAL HEAT PUMP BOREFIELD DESIGN

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**Abstract:** Ground-coupled heat pump (GCHP) systems very often employ vertical borehole heat exchangers (BHE) which are in charge of extracting or injecting heat from and to the ground by exploiting the conductive properties of the soil. The ground-to-BHE-field thermal interactions are a complex transient phenomenon that requires the knowledge of the building energy demand in time and a suitable engineering model for predicting the ground temperature variations in the short and long term. A computationally efficient way to tackle this problem is the recursive calculation of a basic thermal response factor for given different heat pulses representing the building energy demand. In this paper a review of the existing response factor models for BHE analysis is performed and the Finite Line Source (FLS) model is employed to develop and refine new fully analytical and explicit FLS solutions suitable for fast spatial and temporal superposition. The calculations of the temperature response functions are made for a great number of BHE configurations, including different layouts, from compact geometries (e.g. rectangular matrixes) to in-line and open arrangements. Comparisons are finally made among the reference and approximated solutions, also in terms of calculation time. This analysis shows that present analytical expressions can reduce the computation time of large borefield g-function generation down to 1% while maintaining an acceptable average error of about 3% with respect to literature analytical FLS solutions.

**Key Words: ground coupled heat pumps, borehole heat exchanger design, temperature response factors, g-functions.**

## 1 INTRODUCTION

As is well known, ground coupled heat pumps (GCHP) are probably the most energy efficient solution for building space conditioning, especially in heating mode. This technology has encountered a wide diffusion in northern countries, both in Europe and America. GCHP can cover a wide range of energy demand situations, from small residences to large commercial buildings. High efficiencies in such installations are related to the correct design of the borehole field and to the availability of suitable models to simulate the short to long time behaviour of the ground volume associated with the ground heat exchangers. Closed loop heat exchangers are the most common solution for extracting/injecting heat to the ground. Closed loops can be classified into the near horizontal ones and vertical arrangements of borehole heat exchangers (BHE). Vertical heat exchangers are usually preferred to near horizontal or trench counterparts due to the reduced requirement of land surface, the availability of reliable drilling equipment, and to the more favourable ground temperatures at their typical depths. In addition a more consolidated theory has been developed for the BHE design, which is based on the assumption of a pure conduction heat transfer in the soil, condition which is generally a satisfied in the deep ground, where water circulation can be often neglected.

The borefield design goal can be summarized in the definition of the best BHE geometry (with respect to land availability and drilling/connection strategies) and the minimum overall length of vertical pipes. The constraints of the problem and its input information are the building thermal energy demand over time, the ground thermal properties and a target heat pump performance behaviour, in terms of COP, EER or SPF depending on the case.

In order to solve the above described problem, a number of solutions of the transient Fourier problem have been proposed in order to evaluate the ground temperature field when a constant heat flux condition is imposed at BHE boundary.

Typically the borehole is modelled as a linear or cylindrical source, of finite or infinite length. The most popular solutions for such a problem are the so called infinite linear source (ILS, Kelvin, and later by Ingersoll et al., 1954) and infinite cylindrical source (ICS, Carslaw and Jaeger, 1947). Both solutions (Temperature Response Factors, TRF) allow the temperature distribution in the ground to be evaluated in terms of a dimensionless time and distance (radius) from heat source axis. The two solutions can be proved to be in absolute agreement except for the very early times, as also discussed in recent papers (Philippe et al. 2009, Lamarche 2010). Thanks to the work of the Lund research group (e.g. Eskilson, 1987), the TRF approach was extended to the description of complex BHE systems, constituted by finite heat sources arranged in regular arrangements. The Lund approach was based on the numerical solution of the single (finite) heat source problem and on proper superposition techniques in space. The Lund group named the new response factors “g-functions” and a great number of BHE geometries were investigated.

Following the Eskilson contribution, a number of authors successfully applied the response factor approach and the superposition techniques to generate semi-analytical g-functions, thus avoiding to perform time consuming and challenging numerical solutions of the discretized Fourier equation (Yavuzturk and Spitler, 1999, Bernier et al., 2004, Sheriff, 2007, Cauret and Bernier. 2009).

Lamarche and Beauchamp (2007), after Zeng et al. (2002), developed a new solution for the finite line source model (FLS), and provided new possibilities for spatial superposition. Bandos et al. (2009) proposed new analytical and explicit expressions for calculating the ground temperature field induced by a finite line source. Javed and Claesson (2011) more recently rearranged the mathematical development of the FLS solution in a way able to provide new expression which greatly reduces the computation time of the response factor values. Finally Cimmino and Bernier (2013) employed the Javed expressions for superposing the single FLS solution in time and space in order to generate almost constant temperature (in time) g-functions, hence describing a problem similar to the original one tackled by the Lund group.

In spite of the improvements introduced by the above studies in the calculation of the temperature field generated by a single finite length heat source, the superposition of solutions for g-function generation is still a demanding task from a computational point of view, especially if the temperature response factors have to be generated during the design process (i.e. they are not already stored as precalculated values). On the other hand, a fully analytical approximated FLS solution would allow, real time, g-function calculation, suitable for example for fast custom BHE configuration analysis (i.e. non regular BHE arrangements) or web based application development.

In this paper the ILS, ICS, FLS models and the superposition theory are discussed. The FLS solution itself is employed to develop and refine new fully analytical and explicit FLS solutions suitable for fast spatial and temporal superposition and hence for generating thermal response factors for arbitrary BHE arrangements. The base expressions for the new solutions are those proposed by Bandos et al., that unfortunately do not provide satisfactory results when superimposed in time and space. The new simplified solutions are the refined versions of those proposed by Fossa in (2011a, b), here calculated through an optimum search of constants coupled with the comparison with FLS generated g-functions, referring to square and rectangular matrixes, in-line fields and L-shaped BHE configurations.

## 2 FINITE LINE SOURCE THEORY AND PROCEDURE

The thermal interaction between the ground and a BHE field, provided that underground water circulation can be neglected, is governed by the three-dimensional time-dependent conduction equation. Due to the complexity to solving this equation numerically, a number of one-dimensional (in the radial direction) and two-dimensional (radial and axial) analytical solutions have been proposed, able to simulate the ground response to a single constant heat pulse. As is well known, these base solutions (temperature response factors) can be superposed in time and space for describing the transient response of a BHE borefield/ground assembly to any stepwise function describing the variable thermal load to the ground during the seasons.

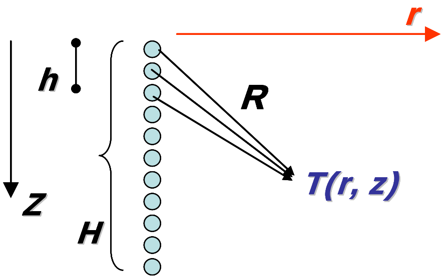
The ILS theory and the related analytical expression are based on the integration over a line of the point source solution (Ingersoll et al., Eq. 1):

$$T(r,\tau) - T_{gr,\infty} = \frac{\dot{Q}}{4\pi k_{gr} r} erfc\left(\frac{1}{2\sqrt{Fo_r}}\right) \tag{1}$$

In the above expression, r is the radial distance from the point source, $k_{gr}$ is the ground thermal conductivity, $T_{gr,\infty}$ is the undisturbed (initial) ground temperature, $Fo_r$ is the radius based Fourier number and finally $\dot{Q}'$ is the heat transfer rate.

With reference to Figure 1 coordinate system, the point source theory can be extended to describe the effects of a finite length line source in an infinite medium as spatial superposition of effects. The resulting expression is Eq. (2):

$$T(r,z,\tau) - T_{gr,\infty} = \frac{\dot{Q}'}{4\pi k_{gr}} \int_0^H \frac{erfc\left(\frac{\sqrt{(z-h)^2+r^2}}{2\sqrt{\alpha\tau}}\right)}{\sqrt{(z-h)^2+r^2}} dh \tag{2}$$



$$R = \sqrt{(z-h)^2 + r^2}$$

**Figure 1 Coordinate reference for integrating the point source solution over a finite line in a infinite medium**

which now includes the heat transfer rate per unit length and the source length H. The next step is to introduce a mirror finite line source in order to simulate a semi-infinite medium. This approach (the FLS hereafter) was first described by Eskilson and later by Zeng et al. (2002). A further contribution in modelling the FLS has been given by Lamarche and Beauchamp (2007), who proposed a particular solution based on the estimation of the ground temperature in time and radius as the average along the vertical coordinate. The new solution involves the complementary error function (erfc) according to the expression:

$$T_{ave}(r,\tau) - T_{gr,\infty} = \frac{\dot{Q}'}{2\pi k_{gr}} \left[ \int_{\beta}^{\sqrt{\beta^2+1}} \frac{erfc(\gamma_F z)}{\sqrt{z^2-\beta^2}} dz - D_A - \int_{\sqrt{\beta^2+1}}^{\sqrt{\beta^2+4}} \frac{erfc(\gamma_F z)}{\sqrt{z^2-\beta^2}} dz - D_B \right] \qquad (3)$$

where $\beta$ is the radial distance made dimensionless by the BHE depth H, and $\gamma_F$ is a function of the square root of the Fourier number based on the BHE depth, $Fo_H$. Equation (3) is especially suitable for further superposition is space in order to describe the behaviour of parallel BHEs through the generation of the corresponding g-functions. Unfortunately Eq. (3) contains improper integrals, to be managed through apposite numerical techniques, which precludes a fast calculation of the g-functions, especially for large (in number) borefields.
More recently Claesson and Javed (2011) reformulated the FLS theory according to new expressions not involving the improper integral in the form of Eq. (2) and including the multiple evaluation of the erf function:

$$T_{ave}(r,\tau) - T_{gr,\infty} = \frac{\dot{Q}'}{4\pi k_{gr}} \left[ \int_{\sqrt{1/4Fo_H}}^{\infty} \exp\left[-(r/H)^2 z^2\right] \cdot \frac{Y(z, D/H \cdot z)}{z^2} dz \right] \qquad (4)$$

$$Y(x,y) = 2ierf(x) + 2ierf(x+2y) - ierf(2x+2y) - ierf(2y) \qquad (5)$$

$$ierf(U) = \int_o^U erf(v)dv = U\, erf(U) - \frac{1}{\sqrt{\pi}}\left(1 - e^{-U^2}\right) \qquad (6)$$

The equation set (4-6) also include the “buried depth” D, which represents the distance from the ground surface of the line source of length H. It also be considered the adiabatic (top) part of a BHE having an overall length of (D+H). This additional parameter, already conceived by Eskilson, has been introduced in the Javed-Claesson formulation by Cimmino and Bernier (2013) in a recent paper.

Worth noticing, Eq. (3) and Eqs. (4-6) lead to the same results in terms of g-function values when D is set to zero (i.e. the BHE is thermally active since the ground top surface).

The simplifications in the FLS model (Eq. 4-6), with respect to the Zeng/Eskilson expressions, allowed a faster evaluation of the temperature field around the single finite source. Nevertheless the g-function generation through spatial superposition requires multiple calculations of the integrals present in Eq. (4-6) and the procedure is still too time consuming, at least when real time or online computations are the goal of a BHE design procedure. This could be the case for a web based application able to calculate (in real time) the g-function for any BHE geometry and then applying the time superposition for managing the variable heat loads to the ground based on the building heating and cooling requirements.

The aim of the present study is to find fully analytical expressions for the finite line source model that can be applied for fast evaluations of any required g-function by spatial superposition. In the following, as done in a previous paper by (Fossa, 2011a, b) the solutions proposed will be referred as AFSS (Approximate Finite line Source Solution).

This investigation is based on a constant refining procedure starting from base formulas able to take into account the independent variables of the problem, namely $Fo_H$, r/H. Constant optimization is in turn based on the comparison with FLS results.

The base formulas selected were those proposed by Bandos et al. (2009), who derived TRF expressions for the average ground temperature around a finite line source (equations 16a and 16b in their paper, “mean T” solutions).

As stressed by Fossa (2011), unfortunately the analytical solutions proposed by Bandos et al. are not directly applicable for spatial superposition and g-function calculation. In particular it can be demonstrated (by comparison with FLS data) that the “mean T” expressions are not suitable for describing the far field (from borehole periphery say for r/H>>rb/H) ground temperature, especially at dimensionless times $\ln(9Fo_H)<0$, where the error can exceed 100%. The work done in this paper is hence to derive, by an optimum search, new constants for an analytical expression that preserves the structure of the Bandos et al. formulas, but can correct the function undesirable features. The new constants were evaluated by applying the spatial superposition to either the “exact” FLS solution or the new approximated ones. With respect to previous work, parameter search is here based on a much wider data set of g-function points and the number of constants involved in optimization was set to N=12. Ad hoc libraries were written and embedded in the Excel spreadsheet in order to either calculate the “true” FLS g-functions or iterate on constants according to the Complex algorithm (Box, 1965). In such a way the TRFs for $M_1$=600 BHE configurations and for 11 g-function points each, were obtained and “optimized” constants derived. Once the best constants have been obtained, further validation have been carried out, by comparing the AFSS g-function values with the corresponding FLS ones for an overall number of 48600 points (600 BHE configurations times 81 g-function values each, Figure 2). All the calculations have been made by setting the buried depth D to zero.

## 3 RESULTS AND DISCUSSION

In order to develop a fully analytical TRF able to describe the H-averaged temperature distribution in time and radial distance, a constant refinement process was applied to a spatial superposition procedure. This choice is better than simply compare the temperature

**Table 1 BHE configuration set for constant refinement**

| **BHE arrangements**<br>B/H=0.03, 0.05, 0.075, 0.1, 0.125<br>$r_b$=0.05 m | |
|---|---|
| Square | 20x10, 20x8, 20x6, 20x5, 20x4, 20x3, 20x2, 18x10, 18x8, 18x6, 18x5, 18x4, 18x3, 18x2, 16x10, 16x8, 16x6, 16x5, 16x4, 16x3, 16x2, 14x10, 14x8, 14x6, 14x5, 14x4, 14x3, 14x2, 12x10, 12x8, 12x6, 12x5, 12x4, 12x3, 12x2, 10x9, 10x8, 10x6, 10x4, 10x3, 10x2, 9x8, 9x7, 9x6, 9x5, 9x4, 9x3, 9x2, 8x7, 8x6, 8x5, 8x4, 8x3, 8x2, 7x6, 7x5, 7x4, 7x3, 7x2, 6x5, 6x4, 6x3, 6x2, 5x4, 5x3, 5x2, 4x3, 4x2, 3x2 |
| Rectangular | 10x10, 9x9, 8x8, 7x7, 6x6, 5x5, 4x4, 3x3, 2x2 |
| In Line | 10x1, 9x1, 8x1, 7x1, 6x1, 5x1, 4x1, 3x1 |
| L shaped | 10x10, 9x9, 8x8, 7x7, 6x6, 5x5, 4x4, 3x3, 10x8, 10x6, 10x4, 10x3 |
| O shaped | 10x10, 9x9, 8x8, 7x7, 6x6, 5x5, 4x4, 3x3, 10x8, 10x6, 10x4 |
| U shaped | 10x10, 9x9, 8x8, 7x7, 6x6, 5x5, 4x4, 3x3, 10x8, 10x6, 10x4 |

values from single finite line sources, since it is possible to better cope with the propagation of errors typical of the superposition techniques.

The process for inferring the final AFSS expressions was based on the following steps. First of all the Complex optimization was applied to all BHE configurations, which include square and rectangular configurations (up to 20x10 BHEs), in-line (up to 10x1), L configurations (up to 10x10). Table 1 summarizes the set of BHE fields here considered. For each borefield, $M_2$=11 g-function values have been calculated in the range of Fourier numbers defined by the limits $-6<\ln(9Fo_H)<4$. The rb/H ratio was kept to 0.0005. Optimization was done by searching the minimum of the function:

$$F(a_1, a_2, .., a_k, .., a_N) = \frac{1}{(M_1 \cdot M_2)} \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} \left| \frac{g_{FLS,j} - g_{AFSS,j}(a_k)}{g_{FLS,j}} \right| \tag{7}$$

where $M_1$ represents the borefield configurations and $M_2$ the g-function values evaluated for each of them.

Validation is then done by extending the number of g-function points for any given configuration set from $M_2$=11 to $M_2$=81.

The expressions selected for such an optimum analysis are the following:

$$T_{ave}(r) - T_{gr,\infty} = \frac{Q'}{2\pi k_{gr}} \left\{ a_1 \left\{ -\gamma + \ln\left[ a_2 \left(\frac{r}{H}\right)^{-2} Fo_H \right] + a_3 \frac{r}{H} + \frac{a_4}{\sqrt{\pi}} \left( \sqrt{a_5\, Fo_H} + \left(\frac{r}{H}\right)^2 \sqrt{\frac{1}{4\, Fo_H}} \right) \right\} \right\} \tag{8a}$$

$$T_{ave}(r) - T_{gr,\infty} = \frac{Q'}{2\pi k_{gr}} \left\{ a_6 \left( a_7 + a_8 \ln\left(\frac{r}{H}\right) + a_9 \frac{r}{H} + a_{10} \left(\frac{r}{H}\right)^2 + \frac{1}{a_{11}\sqrt{\pi}} \left(\frac{1}{Fo_H}\right)^{a_{12}} \right) \right\} \tag{8b}$$

where $\gamma$ is the Euler's constant.

The resulting constant values are reported in Table 2.

For the calculation of the g-function values, by spatial superposition of the present solutions, the highest value between those obtained by Eqs. (8a) and (8b) has to be considered. As a rule of thumb, equation (8a) is to be applied for ln(9Fo$_H$) range from -6 to 0.5 and equation (8b) elsewhere.

It is worthwhile noticing, the functions expressed by Eqs. (8a) and (8b), can assume negative values at very large r/H values and they have to be set equal to zero in such cases, as needed by the physics of the problem (by definition the g-function is a dimensionless temperature excess and cannot assume negative values).

Figure 2 shows the calculated dimensionless excess temperature at BHE periphery, $(T_{ave}(rb)–T_{gr,\infty})/(2\pi k\dot{Q}')$ or g-functions, as calculated with Eqs. (8a) and (8b), by means of spatial superposition. The above approximate g-function values are plotted against the “true” values calculated by means of the FLS solution.

The comparisons cover the ln(9Fo$_H$) range (-6 to 4) and the B/H values from 0.03 to 0.125. The borehole radius to depth ratio $r_b$/H is set to 0.0005 and the buried depth D is set to zero. These range boundary values have to be considered the limits of validity of the proposed AFSS solutions.

**Table 2 Improved constant data set**

| **a1** | **a2** | **a3** | **a4** | **a5** | **a6** |
|---|---|---|---|---|---|
| 0.4749 | 6.6398 | 3.3947 | -3.1378 | 5.6362 | 0.4853 |
| **a7** | **a8** | **a9** | **a10** | **a11** | **a12** |
| -1.9733 | -1.9714 | 3.1427 | -0.9544 | -12.854 | 1.1928 |

Figure 2 contains all the points of the complete data set (48600 g-function values). The linear regression on all data yielded a slope of the fitting line equal to 0.9975, a standard error of the estimate equal to 0.991 and a correlation coefficient equal to 0.999. The average absolute error finally became equal to 3.06%. Figure 3 is the corresponding one where the set of constants proposed in (Fossa 2011b) has been employed. It is apparent the improvement in prediction capability by the present constant set (Table 2).

In order to prove the capabilities of the proposed fully analytical solutions with respect to the literature FLS models, Table 2 compares the computational time requested by the AFSS equations to the corresponding ones from the applications of Eq. (3) and (4-6) for calculating a g-function profile pertaining a 10x10 square configuration. The computational time for FLS, Eq. (3), is set equal to the unity as reference value. Even if the computational time for standard FLS solutions is affected by the choice of the convergence criteria for improper integral routines, the figures in Table 3 show that the proposed solution can dramatically reduce the calculation time (up to 1/30), as requested for example in real time and web based applications.

A final consideration has to be made with reference to the choice to refer to the FLS solutions instead of some numerical data set (i.e. the Eskilson solutions or their recent approximations by Cimmino and Bernier, 2013). First of all, no complete and official g-function data set is available from the Lund group. Furthermore the recent approaches (including also the Monzo et al. one, 2014) are no longer recognised as the most representative TRF scheme for long term analyses of BHE fields. Hence, to Authors’ opinion, FLS has to be considered right now the correct reference for any superposition scheme, in space and time, to be devoted to g-function generation.

**Table 3 Runtime comparison among FLS solutions and present AFSS one, 10x10 rectangular borefield (B/H=0.05)**

| | FLS (Eq. 3) | FLS (Eqs. 4-6) | AFSS |
|---|---|---|---|
| Normalized Runtime[-] | 1 | 0.25 | 0.006 |

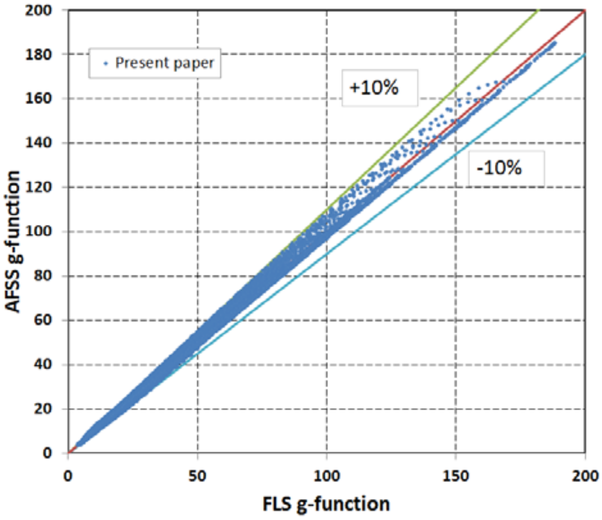


**Figure 2 Comparison between AFSS and FLS g-function values. Present paper constant set**

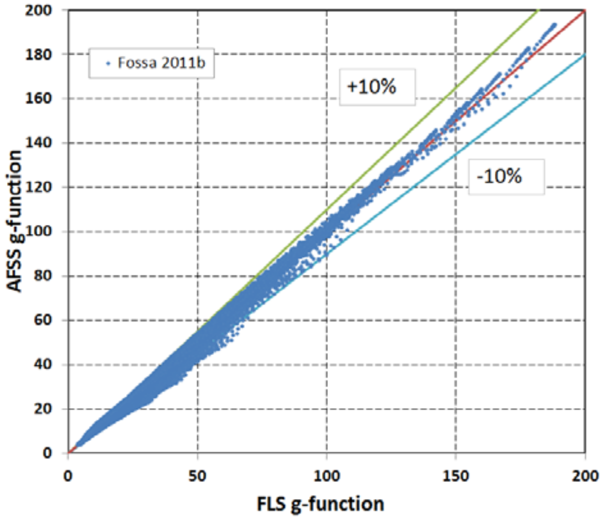


**Figure 3 Comparison between AFSS and FLS g-function values. Constant set proposed by Fossa (2011b)**

## 4 CONCLUSIONS

In this paper the design problem of a system of BHE has been discussed, with special attention to the available solutions for the finite length source problem. This paper focuses on the research of a fully analytical TRF expression to be employed for reliable and fast g-function evaluations. The solution proposed has been derived through an optimization process aimed at refining published formulas. The benchmark here considered was the FLS solution, superposed in space for describing a huge number of borehole configurations. The new formulas have been validated against 48600 g-function values (pertaining to either different geometries or different Fourier numbers). It has been demonstrated that the approximate response factors and the "true" FLS ones are in very good agreement (mean error is 3.06%), and that the use of the present expressions can reduce the computation time for generating any g-function down to 1% with respect to use of literature "exact" FLS formulas. Further work on this subject could include the extension of the current constant set (12 constants) to an enlarged one (18 constants), a constant search analysis based on a different set of reference g-functions (e.g those based on a constant temperature as boundary conditions or referring to a given dimensionless buried depth D/H).

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## REFERENCES

Abramovitz M. and Stegun I.A., 1964. Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables, NBS Applied Mathematics Series 55, National Bureau of Standards, Washington, DC.

Bandos T.V., A. Montero, E. Fernández , J.L.G. Santander, J.M. Isidro, J. Pérez, P.J. Fernández de Córdoba and J.F. Urchueguí, 2009. Finite line-source model for borehole heat exchangers: effect of vertical temperature variations, Geothermics, 38, 263–270.

Bernier M., Pinel P., Labib R. and Paillot R., 2004. A Multiple Load Aggregation Algorithm for Annual Hourly Simulations of GCHP Systems. Int. J. Heating, Ventilating, Air-Conditioning and Refrigeration Research, 10, 471-488.

Box M.J., 1965. A New Method of Constrained Optimization and a Comparison with Other Methods, The Computer Journal 8 (1), 42-52.

Cimmino M., Bernier B., Adams F., 2013. A contribution towards the determination of g-functions using the finite line source, Applied Thermal Engineering, 51, 1–2.

Carslaw H.S.,and Jaeger J.C., 1947.Conduction of Heat in Solids, Claremore Press, Oxford, U.K.

Cauret O. and Bernier M., 2009. Experimental Validation of an Underground Compact Collector Model, Proc. 11th Int.Conf. Effstock Stockholm.

Claesson J., and Javed S., 2011. An Analytical Method to Calculate Borehole Fluid Temperatures for Time-scales from Minutes to Decades. ASHRAE Transactions, 117.2.

Eskilson P., 1987. Thermal Analysis of Heat Extraction Boreholes. Ph.D. Thesis, Lund University of Technology, Sweden.

Fossa M., 2011a. The Temperature Penalty Approach to the Design of Borehole Heat Exchangers for Heat Pump Applications, Energy and Buildings, 43, 1473-1479.

Fossa M., 2011b. A Fast Method for Evaluating the Performance of Complex Arrangements of Borehole Heat Exchangers , Ashrae Hvac & Res. J. vol. 17:6, p. 948-958.

Ingersoll L.R., Zobel O.J., and Ingersoll A.C., 1954. Heat Conduction with Engineering, Geological, and other Applications, McGraw-Hill, New York.

Lamarche L. and Beauchamp B., 2007. A New Contribution to the Finite Line-Source Model for Geothermal Boreholes, Energy and Buildings, 39, 188-198.

Lamarche L., 2009. A fast algorithm for the hourly simulations of ground-source heat pumps using arbitrary response factors, Renewable Energy, 34, 2252-2258.

Lamarche L., 2013, Short-term Behavior of Classical Analytic Solutions for the Design of Ground-source Heat Pumps, Renewable Energy 57 171-180.

Monzo P., Acuna J., 2014. A Novel Numerical Model for the Thermal Response of Borehole Heat Exchanger Fields, 11th IEA Heat Pump Conference, Montréal (Québec), Canada.

Philippe M., Bernier M. and Marchio D., 2009. Validity ranges of three analytical solutions to heat transfer in the vicinity of single boreholes, Geothermics, 38, 407–413.

Sheriff F., 2007. Génération de facteurs de réponse pour champs de puits géothermiques verticaux, M.Sc.A., Thesis, Département de Génie Mécanique. École Polytechnique de Montréal.

Yavuzturk C. and Spitler J.D., 1999. A Short Time Step Response Factor Model for Vertical Ground Loop Heat Exchangers. ASHRAE Transactions, 105, 475-485.

Zeng H.Y., Diao N.R. and Fang Z.H., 2002. A Finite Line-Source Model for Boreholes in Geothermal Heat Exchangers, Heat Transfer-Asian Research, 31, 558-567.