

A SIMPLE TOOL FOR SIMULATION OF GROUND SOURCE HEAT PUMP SYSTEMS

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Abstract: The Building and Environmental Thermal Systems Research Group (BETSRG) at Oklahoma State University has developed a number of ground heat exchanger models for use in simulation of ground source heat pump systems in the HVACSIM+ simulation environment. Unfortunately, HVACSIM+ remains difficult for practicing engineers to use. Nevertheless, there are many situations where an easy to use hourly simulation of a ground source heat pump system would be useful. Therefore, we have developed Excel interfaces to two vertical ground heat exchanger models previously developed in HVACSIM+. At present, this allows users to simulate a ground source heat pump system on an hourly time step for multiple years. This flexibility and ease-of-use comes at the cost of some computation speed – we adopted an iterative scheme to account for the simultaneous effects of the ground heat exchanger on the heat pump and the heat pump on the ground heat exchanger. Nevertheless, we believe this to be a highly useful tool for practitioners who may not have expertise in the use of HVACSIM+, TRNSYS or other similar tools.

Key Words: ground source heat pump system, ground heat exchanger, heat pump, HVACSIM+

1 INTRODUCTION

HVACSIM+ (Park, et al. 1985) is a powerful computer simulation tool first introduced by the National Institute of Standards and Technology (NIST) in 1985. It is intended for use in simulating heating, ventilation and air conditioning (HVAC) systems, as well as controls and the building envelope. The Building and Environmental Thermal Systems Research Group¹ at Oklahoma State University has developed a number of models in the HVACSIM+ simulation environment for use in simulation of ground source heat pump (GSHP) systems. These include models of vertical ground heat exchangers (Xu and Spitler 2006), horizontal ground heat exchangers, heat pumps, and other components. Combining these models in HVACSIM+ to create simulations of complex systems can be quite useful in predicting their behavior, optimizing designs, testing control strategies, etc. However, despite our efforts to create a simpler interface, HVACSIM+ remains difficult for practitioners to use. Assembling a system model that always converges is often a challenge. It can also be difficult for researchers to use! Developing new models can also be quite time-consuming. Recently, we decided that these problems could be ameliorated with development of a new tool that uses Microsoft Excel and its native programming language, Visual Basic for Applications (VBA) to simulate the heat pumps and other components and uses HVACSIM+ to model the ground heat exchanger. This paper reports on this simplified tool and gives an example of using it to model a GSHP system with a heat pump that utilizes electric resistance backup heat.

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Although the example is for a residential system with backup electrical resistance heating, the tool should be suitable for a wide range of system types, as long as the simulation of the system besides the ground heat exchanger can be simulated in Excel/VBA in a reasonable amount of computational time. The user simply takes the fluid temperature exiting the ground heat exchanger as an input and provides the amount of heat rejected or extracted. The tool will iterate back and forth between the ground heat exchanger model and the user-provided model implemented in VBA. This does rely on the user having some facility with VBA. Features of the tool include:

- The simulation can start and stop on any day of the year; it can simulate multiple years with different loads or multiple years, repeating the same year's loads.
- We have adopted an hourly time step.
- Building hourly heating and cooling loads are calculated with another program such as Energy Plus and treated as inputs here.
- The circulating pump is not explicitly modeled; the user simply specifies the mass flow rate. Type 620 requires users to enter a constant flow rate and Type 621 requires users to enter hourly mass flow rate. However, the user can model pumping power and even include it as additional heat rejection to the ground.
- The ground heat exchanger model requires g-functions. The tool has two approaches for obtaining g-functions; they can be read from a file written by GLHEPRO (Spitler 2000) or they can be computed directly from the tool using the method of Javed and Claesson (2011).
- The user specifies the following information about the ground heat exchanger; the file written by GLHEPRO will contain most of this information.
 - The borehole configuration – number of boreholes, arrangement, and spacing.
 - The ground thermal properties – thermal conductivity, volumetric heat capacity, and undisturbed temperature.
 - The fluid type and % by weight antifreeze if antifreeze is used.
 - Information about the borehole completion – type of heat exchanger, pipe and grout thermal properties, U-tube position in borehole.
- During system operation, the ground heat exchanger and heat pump affect each other. The amount of heat that the heat pump rejects or extracts is affected by the entering fluid temperature to the heat pump; the exiting fluid temperature of the ground heat exchanger is affected by the amount of heat rejection or heat extraction. Because only the ground heat exchanger model is implemented in HVACSIM+, we have adopted an iterative scheme to ensure that the entire simulation converges. This gives us simplicity, flexibility and robustness at the expense of computational efficiency. We think this is an acceptable tradeoff.

The tool has recently been used to perform several studies (Gehlin and Spitler 2014, Spitler, et al. 2014) looking at over 100 different ground heat exchanger designs for residential GSHP systems in Sweden. Swedish residential heat pumps have several features that are different from commonly used GSHP in North America: combined house heating and domestic water heating; integrated hot water storage; and condenser temperature control based on outdoor air temperature combined with a control curve; prioritization of domestic hot water heating; and backup electric resistance heating. Use of the tool allowed rapid implementation of the heat pump model and it was relatively easy to automate the simulations so that 121 simulations with different heat pump sizes and borehole depths could be run automatically. Then, the hourly results for a five-year period used in the study could be readily post-processed with some additional VBA code combined with the Excel spreadsheets.

2 METHODOLOGY

Figure 1 shows an overview of how an entire analysis might be done with the tool. The dashed lines represent divisions between different programs. Above the top dashed line is the building simulation program used to calculate the hourly heating and cooling loads on the heat pump or heat pump system. We've used EnergyPlus (US DOE 2012), but there are many such programs. Between the two dashed lines is the Excel/VBA tool; it takes user inputs entered into Excel sheets and can also calculate the g-functions for the ground heat exchanger model or read them from a file produced by GLHEPRO. The Excel/VBA tool executes HVACSIM+ and automatically iterates to find a converged solution. The box labelled "Compute hourly heat rejection/extraction rates" represents either a model of a heat pump or a heat pump system, possibly containing multiple heat pumps and/or supplemental heat rejection or extraction. This part would be coded by the end user.

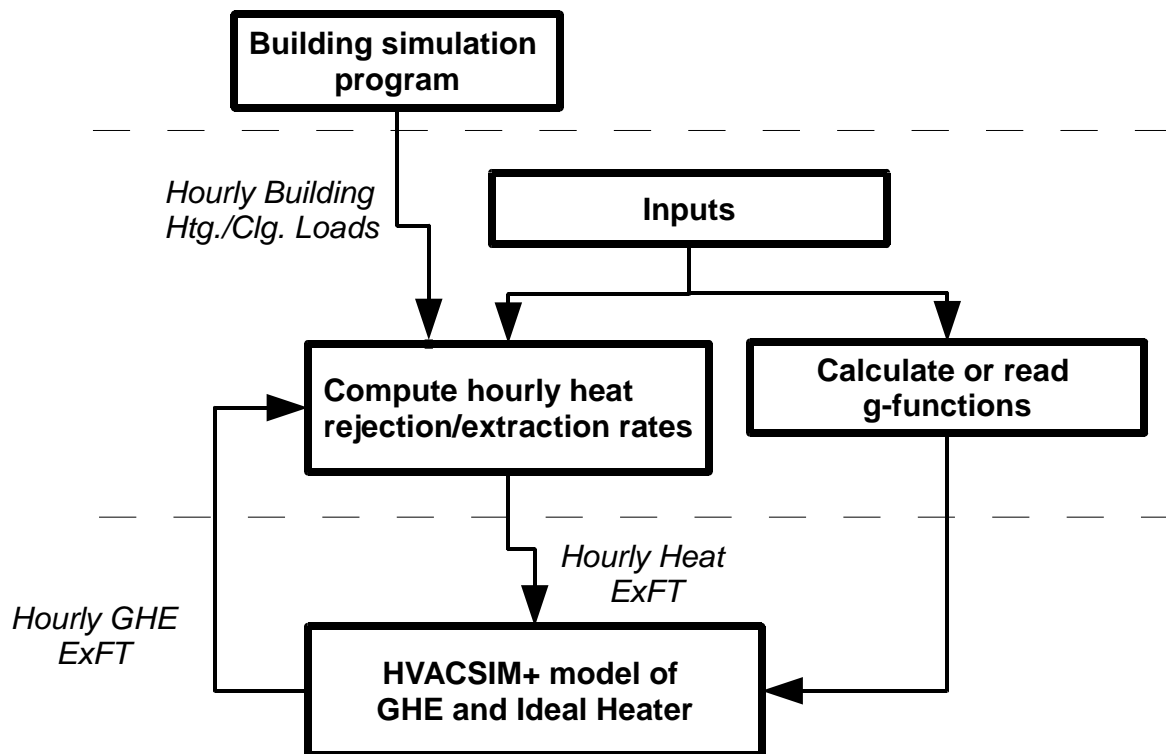


Figure 1: Schematic of the Ground Source Heat Pump System

In order to simulate the entire system, a minimum of two component models are needed - the ground heat exchanger model and the heat pump or heat pump system model. However, in order to model the system as we have done here, a third component model, the "ideal heater" is needed within HVACSIM+. Two unknowns, the ground heat exchanger exiting fluid temperature ($ExFT$) and the heat pump source side heat transfer rate q_s , can be solved by employing the two models iteratively. Were the heat pump and ground heat exchanger both modeled within HVACSIM+ or TRNSYS, the two variables would be solved iteratively at each time step. We have taken a simpler approach – starting with initial guesses for ground heat exchanger $ExFT$ for the entire duration of the simulation, we calculate with the heat pump model the source side heat transfer rate on an hourly basis for the entire duration of the simulation. These source side heat transfer rates are used by the ground heat exchanger model to find new estimates of the ground heat exchanger $ExFT$. This iteration between two models, each operated for the entire duration of the simulation, is robust – it seldom fails to converge – but not particularly fast. On a 2013-era desktop PC, simulation of

the ground heat exchanger and the heat pump with backup electric resistance heating described below takes about 55 seconds for a 2-year simulation and 145 seconds for a 5-year simulation.

2.1 Heat Pump Model

The box labeled “Compute hourly heat rejection/extraction rates” in Figure 1 could represent anything from a simple equation fit model to a model that combines supplemental heat rejection or extraction. Here, we describe a simple equation fit model of a water-to-air heat pump model with backup electric-resistance heating as might be used in the northern part of North America. (But, from our perspective, the point of the tool is to allow the user to readily implement alternative system configurations.) For this model, the heating capacity of the heat pump is determined with an equation fit. The electric resistance heating will usually be controlled with a two-stage thermostat and the electric resistance heating will be activated when the temperature falls too far below the setpoint. Our model approximates this by activating the electric resistance heating any hour the heating demand exceeds the available heat input. The model, which is an extended version of the model described by Spitler(2000) may be briefly described as follows for a single hour of heating operation.

The heating capacity is fit, for the intended source-side flow rate and load-side conditions, as a 2nd order polynomial function of source-side entering fluid temperature, EFT_s :

$$q_{cap,htg} = a_1 + a_2 EFT_s + a_3 EFT_s^2 \quad (1)$$

The hourly run-time fraction is computed as the ratio of the hourly load to the hourly capacity.

$$RTF = \frac{q_{load,htg}}{q_{cap,htg}} \quad (2)$$

If the computed value is greater than one, the load met by the electric resistance heating is then given by:

$$q_{ERH} = (RTF - 1) \cdot q_{cap,htg} \quad (3)$$

And the run time fraction would be set to one.

The heat extraction rate is calculated with the ratio of the heat extracted to the heating provided, which is also a 2nd order polynomial function of the source-side EFT , fit to manufacturer's data:

$$R_{htg} = b_1 + b_2 EFT_s + b_3 EFT_s^2 \quad (4)$$

The heat extraction rate is then:

$$q_{S,htg} = q_{cap,htg} \cdot RTF \cdot R_{htg} \quad (5)$$

The heat pump COP can be determined as:

$$COP = \frac{1}{1 - R_{htg}} \quad (6)$$

And the heat pump electrical power can be determined as:

$$q_{EHP} = \frac{q_{S,htg}}{COP} \quad (7)$$

The coefficients in Equations 1 and 4 would be determined from manufacturer's data. The model for cooling would be very similar, except that there is no possibility in this system for supplemental cooling, so any hour with *RTF* exceeding one would be flagged as an error. An example application of the model is given in the Results section.

2.2 Ground Heat Exchanger Model

The ground heat exchanger model is briefly described in Section 2.2.1. Determination of the g-functions for the model is described in Section 2.2.2. The implementation of the model within an HVACSIM+ simulation is described in Section 2.2.3 and the interface between the Excel/VBA tool and HVACSIM+ is described in Section 2.2.4.

2.2.1 Type 620/621 Ground Heat Exchanger Models

Two ground heat exchanger models implemented in HVACSIM+ are supported by the tool:

- The Type 620 model which has a constant borehole resistance and is intended for situations where the ground heat exchanger flow rate does not vary widely.
- The Type 621 model which has a borehole resistance calculated on the fly and, so, for example, can be used in a case where the flow in the U-tube drops into the laminar regime during the simulation.

Both the Type 620 and Type 621 ground heat exchanger models use response functions, called g-functions, to calculate the temperature response of the ground heat exchanger to a series of heat rejection/extraction rates. The heat extraction and rejection loads on the ground heat exchanger are devolved into a series of step inputs, then the g-function is used to determine the response due to each step input, and the temperature responses are superimposed to determine the evolution of borehole temperatures with time. The method was originally developed by Claesson and Eskilson (1988); see Eskilson(1987) for a more detailed treatment. Yavuzturk and Spitler (1999) extended the method to shorter time steps, as short as, say, 10 minutes. Xu and Spitler (2006) developed the variable borehole resistance approach; see Xu (2007) for more details.

2.2.2 Determination of g-functions

The model supports two methods for determining g-functions. The first method involves using GLHEPRO, which has a database of long-time step g-functions that are combined with short time-step g-functions calculated by GLHEPRO. GLHEPRO writes a data file that provides g-functions and all other parameters needed by the Type 620 and 621 component models in HVACSIM+. The second method is internal to the program (implemented in VBA code) and is based on the work of Claesson and Javed (Claesson and Javed 2011, Javed and Claesson 2011) for calculating g-functions. The Claesson and Javed method treats the boreholes as having uniform heat fluxes rather than uniform wall temperatures as in the Claesson and Eskilson method. The Claesson and Javed method makes it tractable to compute the g-functions on the fly rather than requiring a database. This has several advantages – it allows any pattern of boreholes to be specified, and it allows the Excel/VBA tool to compute the g-functions as the first step in a simulation. This makes it feasible to make parametric studies, e.g. varying the borehole depth repeatedly and automatically. It has the disadvantage (we believe, based on consideration of the physics) of being less accurate than the Claesson and

Eskilson method, particularly as the number and density of boreholes is increased. The differences between the two are investigated by Malayappan and Spitler (2013).

Both methods rely on calculation of the borehole resistance using the multipole method (Claesson and Hellström 2011) as part of the short time-step g-function computation; our implementations are based on the description in Claesson and Bennet (1987) and Bennet, et al. (1987).

2.2.3 Implementation of the HVACSIM+ simulation

The flow of information in the tool starts with hourly heating and/or cooling loads to be met by the heat pump and, if present, supplementary heating or cooling. The system model implemented by the user in Excel/VBA is formulated so that it takes hourly entering fluid temperatures from the ground heat exchanger and returns hourly heat extraction/rejection rates. The model may use other information provided by the user. E.g. the implementation described in Gehlin and Spitler (2014) used the hourly outdoor air temperature along with a control curve to determine the heat pump fluid temperature setpoint. In HVACSIM+, the ground heat exchanger model takes the entering fluid temperature and mass flow rate as inputs and returns exiting fluid temperature. Therefore it is necessary to have one additional component, which we call an “ideal heater” that, within the HVACSIM+ simulation, imposes the hourly heat extraction/rejection rates on the ground heat exchanger. (It’s very “ideal” – it can heat or cool!) The ideal heater (Type 643 in our library) simply takes the hourly heat extraction/rejection rates as a boundary condition, and, given the fluid mass flow rate and properties, determines the change in temperature across the heater that corresponds to the heat extraction or rejection rate.

2.2.4 Interface between the tool and the HVACSIM+ simulation

The tool interfaces to the HVACSIM+ simulation with three text files that are written prior to each iteration:

1. Simtemp.bnd holds the hourly boundary conditions – the loads on the ground heat exchanger and the hourly ground heat exchanger mass flow rates.
2. Inputfile.dat holds the start and stop times, as well as intermediate file names.
3. Simtemp.dfn holds the description of the system – how the two component models are connected, initial conditions, and all parameters, such as the g-functions.

Once the user has pressed the “Simulate” button, the VBA code runs the user-specified heat pump model or heat pump system model to determine the hourly heat rejection/extraction rate; writes the three text files; then executes modsim.exe (the HVACSIM+ simulation executable); modsim.exe writes an output file, called simtemp.out, that holds the hourly exiting fluid temperatures from the ground heat exchanger; the VBA code reads this file, and compares the temperatures to the previous iteration. If the maximum difference is less than 0.1°C, the simulation is considered converged. If not, the above steps are repeated.

3 RESULTS

In this section, we provide sample results using the tool for a case that cannot readily be simulated with an existing HVACSIM+ model – a GSHP system with a water-to-air heat pump and backup electric resistance heating. Such a system will be sensitive to the ground heat exchanger depth – as the ground heat exchanger size is decreased, heat pump entering fluid temperatures will decrease, heat pump capacity will decrease, and there will be increased demand for backup electric resistance heating.

A prototype house located in Sioux Falls, South Dakota was modeled in Energy Plus to determine the hourly heating and cooling loads for a typical weather year. The house has a rectangular plan - 15.24 m X 9.75 m, with a floor area of 148.6 m². The house has a flat roof and is 2.7 m high. 25% of the wall area is covered by glazing on the north and south facades, and 10% of the east and west facades are glazed. The windows have a U-value of 2.5 W/(m²·K) and a solar heat gain coefficient of 0.36, corresponding to a double glazing with a low E coating. The wall and roof are constructed with structural insulated panels with an R-value of 7.4 K/m²·W (R42 in US units). Heating/cooling setpoints are 20°C/23°C respectively. Annual total heating loads are 10,778 kWh; annual total cooling loads are 1478 kWh. The building peak heating load is 6.7 kW and building peak cooling load is 2.5 kW.

A ClimateMaster TSV 024 water-to-air heat pump is chosen for this study; the nominal cooling capacity of the heat pump is 7.0 kW. The heat pump is equipped with a backup electric resistance heater that allows the system to always meet the required hourly heating load. I.e., as modeled, the heating load is always met, and the electric resistance heat required is simply calculated.

The ground heat exchanger consists of four boreholes in a line configuration with a spacing of 4.6 m. The working fluid is a 20% (by weight) propylene glycol-water mixture, with a freezing point of about -8.1°C. One challenge in designing the system is plumbing the boreholes so that turbulent flow at low temperatures can be maintained, and we found that difficult. A compromise solution, labeled “Partially turbulent” below, used the highest flow rate given by the manufacturer in the catalog data, 0.38 L/s, and the four boreholes plumbed so that there are two pairs in parallel; each pair is in series. US nominal size ¾” SDR-11 HDPE tubing with an inside diameter of 21.8 mm is used. This configuration, at 60 m borehole depth and a mean temperature of 0°C, has a calculated pressure drop of 114 kPa or 11.6m of head loss. If the circulator efficiency is 25%, the pumping power is 172 W. Even with such high pumping power, as shown in Figure 2, the Reynolds number drops below 2300, the critical value (Incropera and DeWitt 1990) for onset of turbulence and never reaches 10,000, the value thought to be required for fully turbulent flow.

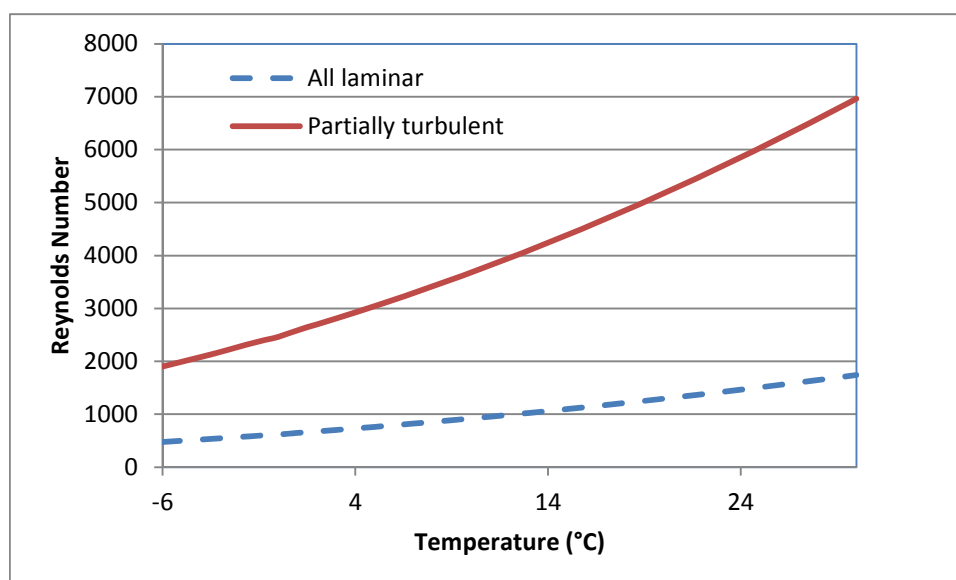


Figure 2: Reynolds number in U-tube for two cases

Given the high pressure drop associated with attempting to reach turbulent flow, a second case “All laminar” was also investigated. For this case, the lowest flow rate given by the

manufacturer in the catalog data, 0.19 L/s, was used and the four boreholes were plumbed in parallel. The same tubing as the previous case was used. This configuration, at 60 m borehole depth and a mean temperature of 0°C, has a calculated pressure drop of 21 kPa or 2.1 m of head loss. If a circulator with efficiency of 25% can be found, the pumping power would only be 16 W.

The system is simulated with four different borehole depths - 40 m to 70 m – and with the two piping designs described above. The ground thermal conductivity is 2.82 W/mK, and the volumetric heat capacity is 2160 kJ/m³K. Standard bentonite grout is used, and the calculated borehole resistance for the “Partially turbulent” case is 0.25 K/(W/m). For the “All laminar” case, the borehole resistance is 0.28 K/(W/m).

Eight cases – four borehole depths and two piping designs – were simulated for a two-year period, beginning on January 1. The 2nd year is chosen for comparison purposes below. The energy consumption results are summarized in Tables 1 and 2. Figure 3 illustrates the monthly breakdown of energy consumption for two different piping configurations with 60 m deep boreholes. As expected, increasing the borehole depth for either configuration leads to more favourable fluid temperatures decreasing the heat pump electrical energy consumption, as well as the electrical energy required for the resistance heat. However, savings in heat pump and resistance heater energy consumption are partly offset by increased pumping energy requirements, so the seasonal coefficient of performances (SCOP) for heating only increase by about 5% for the mostly turbulent case and 8% for the laminar case. For cooling, with the ground heat exchanger dominated by heat extraction, the electrical energy consumption of the heat pump barely changes with borehole depth. For the mostly turbulent flow cases, the cooling SCOP actually drops with increasing borehole depth due to increased pumping energy. For the all laminar cases, the pumping energy has much less influence on the system performance and the heating SCOP improves by 3% going from 40m deep boreholes to 70 m deep boreholes.

Table 1: Electricity consumption of GSHP System – Mostly turbulent flow case

Borehole depth (m)	HP Ann. Elec. Consumption- Heating (kWh)	Resistance Htg. Ann. Elec. Consumption (kWh)	Circ. Pump Ann. Elec. Consumption - Heating (kWh)	Heating SCOP	HP Ann. Elec. Consumption - Cooling (kWh)	Circ. Pump Ann. Elec. Consumption - Cooling (kWh)	Cooling SCOP	Total annual electricity consumption (kWh)
40	3059	64	249	3.20	177	24.9	7.32	3575
50	2977	39	264	3.29	175	27.2	7.33	3482
60	2922	25	281	3.34	173	29.4	7.29	3431
70	2882	16	298	3.37	172	31.7	7.25	3400

Table 2: Electricity consumption of GSHP System – All laminar flow case

Borehole depth (m)	HP Ann. Elec. Consumption- Heating (kWh)	Resistance Htg. Ann. Elec. Consumption (kWh)	Circ. Pump Ann. Elec. Consumption - Heating (kWh)	Heating SCOP	HP Ann. Elec. Consumption - Cooling (kWh)	Circ. Pump Ann. Elec. Consumption - Cooling (kWh)	Cooling SCOP	Total annual electricity consumption (kWh)
40	3228	107	29.3	3.20	206	2.8	7.08	3573
50	3148	69	29.6	3.32	202	2.9	7.20	3452
60	3093	50	30.2	3.40	200	3.1	7.27	3376
70	3052	37	30.9	3.45	199	3.2	7.32	3322

Comparing the partly turbulent flow cases and the all laminar flow cases, the all laminar flow cases actually come out ahead, with energy savings of 0-2% over the partly turbulent flow

cases. This outcome is slightly different than what might be expected from common design recommendations.

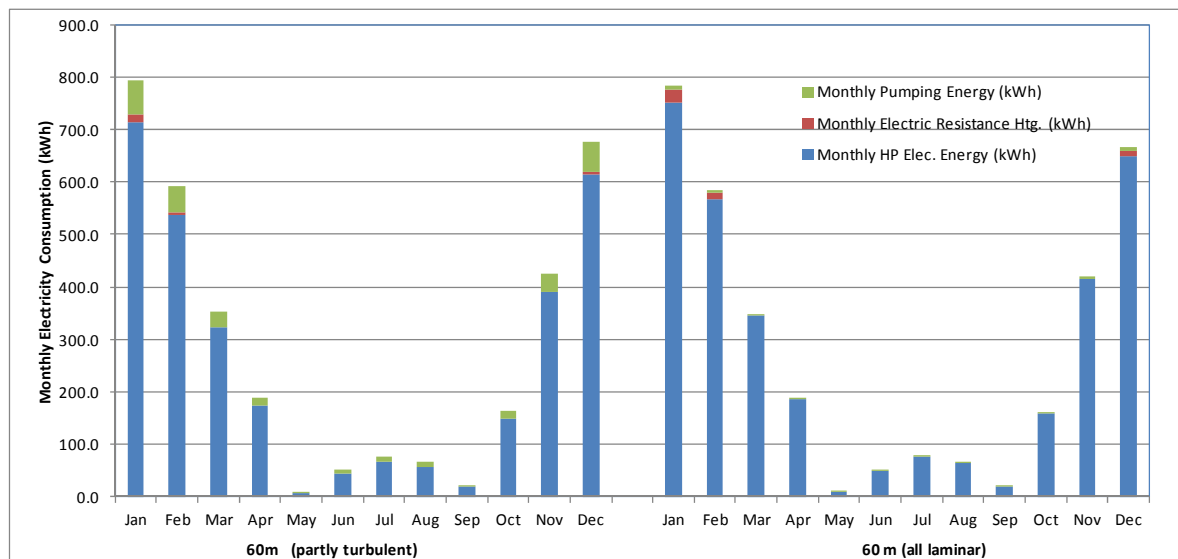


Figure 3: Monthly energy consumption for the two piping configurations, 60 m deep boreholes

4 CONCLUSIONS

This paper has presented an interface between Excel and two ground heat exchanger models developed in HVACSIM+. The Excel/VBA code can be used to model a simple GSHP system, or more complicated systems with backup resistance heating (as illustrated here or by Gehlin and Spitler (2014)), or, we imagine a range of other GSHP systems. For example, with a simple cooling tower model, it should be readily possible to model hybrid ground source heat pump systems. The iterative scheme, which iterates between simulations of the ground heat exchanger and the GSHP system, each of which covers the entire simulation duration, has the advantage of being robust, if not particularly fast.

Use of this tool has been demonstrated for simulation of a residential GSHP system serving a house in Sioux Falls, South Dakota. The GSHP has supplemental electric resistance heating and the overall heating and cooling SCOPs for the system varies with borehole depth and piping configuration. Contrary to expectations, the configuration for which the flow in the boreholes was always laminar used 0-2% less electrical energy than the configuration for which the flow in the boreholes would be partly turbulent.

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