

## **A NEW APPROACH TO THE STUDY OF THE HYDRO-THERMODYNAMIC SYSTEM "GROUND – BOREHOLE HEAT EXCHANGER – GROUND SOURCE HEAT PUMP"**

*Olga Kordas, Senior Researcher, KTH – Royal Institute of Technology, Teknikringen 34 (Stockholm, Sweden);*

*Eugene Nikiforovich, Professor, Institute of Hydromechanics of NASU, 8/4 Zhelyabova street (Kyiv, Ukraine)*

**Abstract:** Increasing energy efficiency of the ground source heat pump (GSHP) systems is an important task for further development and implementation of this renewable energy technology for industrial and domestic heating and cooling. Achieving the higher efficiency, in its turn, requires insight into thermodynamic interaction of all elements of GSHP systems. The presented in the paper mathematical model of a strongly non-equilibrium thermodynamic systems G - BHE - GSHP has been elaborated to describe the energy exchange between the ground, BHE and evaporator of GSHP. Based on this model a stationary problem of such system operation has been solved. In particular, it is shown that the stationary energy exchange processes in the G - U-tube BHE - HP system is characterized by a unique dimensionless parameter - the ratio of the thermal conductivity of the ground and brine. For a stationary case a one-parameter universal dependence of the amount of energy extracted from the ground using U-tube BHE, which is essentially non-linear in length, was obtained. The physical interpretation of the received results is provided.

**Key Words:** strongly non-equilibrium thermodynamic systems, optimal GSHP systems, energy discharge of BHE, similarity and dimensional methods

### **1 INTRODUCTION**

The ground source heat pump (GSHP) is recognized as one of the most effective renewable energy technologies for heating and cooling residential houses and commercial spaces (RHC-PLATFORM 2011). Reliable functioning of this technology demands essential efforts for selection, analysis and optimal utilization of a renewable energy source. Experiences drawn from the past cold winters demonstrate (Moroz et al. 2010) that a geothermal system (GS), which utilizes thermal energy generated and stored in the Earth is the most reliable and effective.

By GS we mean an energy source for a brine/water heat-pump system, which is a heat exchanger in the ground, a so-called energy well. There are horizontal and vertical GSs. Though horizontal GSs are much cheaper than the vertical ones, they occupy a larger area and their energy efficiency (energy extraction capability) depends on the seasonal variability of the temperature field in the near-surface layer of the earth, which results from the interaction between the solar and geothermal energy fluxes. This means that for successful functioning of such a system, its size or the length of the ground heat exchanger (GHE) should be calculated for the coldest period. Vertical GSs do not suffer from this shortcoming. It is a well-known fact (Ochsner 2008) that from a depth of several meters, the temperature of the medium becomes almost constant, thus making a vertical GS (VGS) an almost unlimited, reliable, and energetically stable thermal source.

Though the main limitation of a GS is its high cost, its ability to generate the necessary amount of energy even during cold winters makes this source very attractive.

A VGS consists of ground heat exchangers of various geometries placed in a vertical borehole filled with brine. Various designs of such heat exchangers are described in (Acuna 2013). An essential task in the design of VGS is to calculate the depth of the borehole necessary for generating given amount of thermal energy. To this end, it is essential to consider the thermodynamic interaction of all elements of a GS and a heat pump (HP), which determines the energy exchange among the environment, energy well, and HP evaporator.

There are a great variety of analytic, numerical, and experimental studies of heat transfer in VGS. Analytic models usually regard a borehole as an infinite linear (Ingersol and Plass 1948) or cylindrical (Carlsow and Jaeger 1959) heat source/sink. These studies were further developed in (Deerman and Kavanaugh 1991). Numerical models for the analysis of heat transfer in ground heat exchangers are presented in (Zhou et al. 2006; Zhang et al. 2001; Fan and Ma 2006). It should be noted that the existing models are very complicated and contain many empirical parameters. This hinders the clear interpretation of the physical processes in VGSs. Moreover, as previously mentioned, the existing analytic and numerical models represent a borehole as an infinite linear or cylindrical heat source/sink with constant (along the length) temperature or constant local heat fluxes not related to the energy characteristics of the HP. This allows determining the heat per unit length extracted by the VGS. While, as shown by the recent experimental studies (Acuna 2013), the local heat fluxes are strongly dependent on the depth of the borehole. Hence, for the correct determination of the VGS length, we need to know how heat fluxes are formed along the borehole length. The state of the art in the analytic, numerical, and experimental modeling of VGS is discussed in (Yang et al. 2010).

From the hydrothermodynamic point of view, a GS with a heat pump is a strongly non-equilibrium thermodynamic system with different heat-transfer mechanisms: molecular or convective/molecular in the environment, molecular through the walls of ground heat exchangers, and molecular/convective inside the tubes of the heat exchanger and during phase transitions in the evaporator.

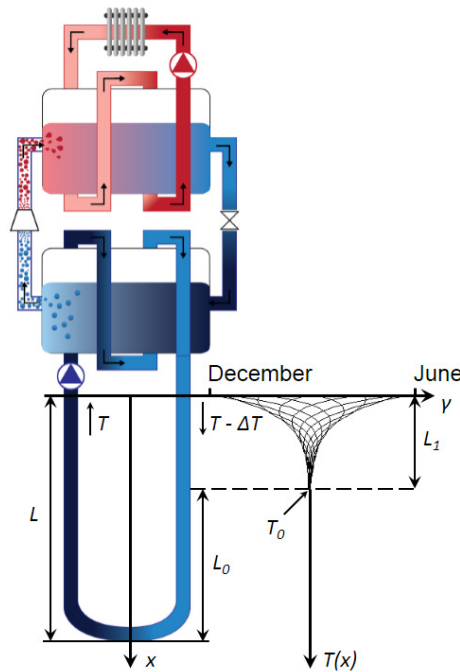
Our objective here is to construct a mathematical model of such a strongly non-equilibrium hydrothermodynamic system with continuous temperature and heat fluxes at the interfaces between its components. The model enables study of formation of energy fluxes and design of an energy-optimal geothermal system with a heat pump by determining the limiting stages of energy exchange between its components and their adjustment.

## 2 PROBLEM STATEMENT

Figure 1 schematizes a vertical geothermal system with a heat pump of capacity  $E$  and a vertical energy well of length  $L$ . Let us consider in more detail the undisturbed temperature field in the upper soil layer, which is a source of renewable energy for the heat pump. The same figure shows, on the right, a typical seasonal variability of the undisturbed temperature  $T_0(z, t)$  in the upper soil layer for  $x \geq 0$ , where  $h$  is the depth and  $t$  is time. An intrinsic feature of the temperature profile in ground is the presence of a seasonal variability layer of length  $L_1 \approx 10 \text{ m}$ , which results from the interaction between the geothermal heat flux from the Earth, the radiation heat flux from the Sun, and the molecular heat flux from the atmosphere. The temperature difference on the ground surface  $x = 0$  in the midlatitudes may reach  $40^\circ\text{C}$ . The temperature at the depth  $x \geq L_1$  remains constant with a low positive gradient on the order of  $3^\circ\text{C}/100 \text{ m}$ . During the cold (or heating) season, the temperature in the upper layer is lower than the undisturbed temperature at the depth  $x \geq L_1$ . Hence, the seasonal variability layer reduces the efficiency of (the amount of energy extracted by) the VGS. The same is true for the summer period when the VGS is used as a passive thermal source for cooling. Therefore, to increase the energy efficiency of the VGS, it is necessary to exclude the upper layer from consideration. Technically, this means thermal insulation of vertical VGSs within this layer. Consequently, the VGS is assumed to be within the layer  $x \geq L_1$  with constant temperature  $T_0$ . The thermal capacity generated in the evaporator of the heat pump can be determined by the formula

$$E_0 = c_p \rho \Delta T \cdot Q \quad (1)$$

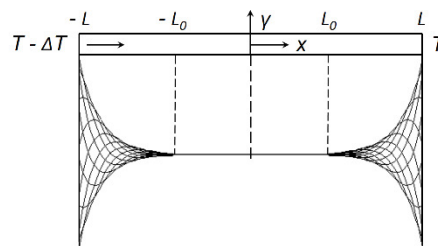
where  $c_p$  and  $\rho$  are the specific heat capacity at constant pressure and density of the brine in the heat exchanger;  $\Delta T$  is the temperature difference between the inlet and outlet of the HP evaporator;  $Q$  is the volume flow rate of the brine in the heat exchanger ( $\text{m}^3/\text{sec}$ ). The magnitude of  $Q$  is obviously determined by the heat-exchanger length and design features of the HP evaporator. Hence, a given capacity of the HP can be provided by finding the length  $L$  of the heat exchanger that provides a given value of  $E_0$  at a certain flow rate  $Q$  of brine.



**Figure 1: Geothermal system with heat pump and seasonal variability of the thermal fields in soil**

In the case of a U-pipe borehole heat exchanger, the statement and solution for the problem of determining the length  $L$  depending on the capacity  $E_0$  of the heat pump involves certain difficulties associated with the complicated geometry of the problem. Since the heat exchanger is placed in a medium over which the undisturbed temperature  $T_0$  is uniformly distributed, we assume that the descending and ascending sections of the exchanger are thermodynamically decoupled (this assumption will be evaluated below).

With this assumption, the extraction of geothermal energy by a U-tube heat exchanger of length  $L$  can be represented as heat exchange between the brine flow  $Q$  in a pipe of radius  $r$  and length  $L$  and the external semi-infinite medium of temperature  $T_0$ . Such a heat exchange process is schematized in Figure 2.



**Figure 2: Scheme of "unbent" ground U-pipe heat exchanger**

In the figure, the points  $x = -L_0$  and  $x = L_0$  are the outlet and inlet of the HP evaporator (i.e., the temperature at these points is equal to the temperature of the brine at the outlet and inlet of the evaporator).

We will use cylindrical coordinates  $(r, z)$  with origin at the middle of the heat exchanger. In this coordinates, the heat exchange between the brine and the environment is described by the following system of equations:

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T_s}{\partial r} = 0, \text{ for } r > R \quad (2)$$

$$V_x \frac{\partial T_b}{\partial x} = \chi_b \left( \frac{\partial^2 T_b}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T_b}{\partial r} \right) \text{ for } r > R \quad (3)$$

where the subscripts  $s$  and  $b$  refer to the soil and the brine, respectively;  $\chi_b$  is the thermal diffusivity of the brine. In real conditions, the brine flow in the heat exchanger is quite slow; therefore, it may be assumed to be a Poiseuille flow:

$$V_x = V_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right], \quad (4)$$

where  $V_{\max}$  is the maximum velocity of Poiseuille flow on the pipe axis ( $r = 0$ ). Let us also assume that the effect of the pipe wall on the heat exchange between the ground and the brine can be neglected. This assumption is not essential for the solution of the problem and it means that the material of the pipe has sufficient heat conductivity.

Let us introduce typical length scales  $L_x$  in the longitudinal direction (because the problem does not include such a length scale, it should be determined by analyzing Eq. (3)) and  $R$  in the transverse direction and define dimensionless variables:

$$r^* = r/R, \quad x^* = x/L_x; \quad (5)$$

Since  $V_{\max} = 2Q/\pi R^2$ , Eq. (3) can be rearranged as

$$(1 - r^{*2}) \frac{\partial T_b}{\partial x^*} = \frac{L_x \cdot \chi_b \pi}{2Q} \left[ \left( \frac{R}{L_x} \right)^2 \frac{\partial^2 T}{\partial x^{*2}} + \frac{1}{r^*} \frac{\partial}{\partial r^*} r^* \frac{\partial T}{\partial r^*} \right] \quad (6)$$

The left-hand side of (6) describes heat transfer by convection in the pipe, and the right-hand side describes heat transfer by conduction. It is obvious that, physically, both convective and molecular heat transfer should be taken into account. Hence, Eq. (6) yields an expression for the characteristic length scale  $L_x$ :

$$L_x = \frac{2Q}{\pi \chi_b}. \quad (7)$$

With (1), formula (7) becomes:

$$L_x = \frac{2E_0}{\pi \lambda_b \Delta T}, \quad (8)$$

where  $\lambda_b$  is the thermal conductivity of the brine. Thus, the characteristic length  $L_x$  explicitly depends on the capacity  $E_0$  of the heat pump and the temperature difference  $\Delta T$  across the evaporator. From the similarity and dimension theory (Sedov 1959) it follows that the length  $L_0$  of the ground heat exchanger must be proportional to  $L_x$ , which provides its functional dependence on the HP characteristics. This dependence will be detailed below.

Let us evaluate the expression  $\left(\frac{R}{L_x}\right)^2$  in Eq. (6). Using (8), we obtain

$$\left(\frac{R}{L_x}\right)^2 = \left(\frac{R \cdot \lambda_b \pi \cdot \Delta T}{2E}\right) \ll 1$$

for the real parameters of the GS. This means that the diffusion of heat in the  $x$ -direction in the brine may be neglected. Then the system of equations (2), (3) becomes:

$$\frac{\partial}{\partial r} r \frac{\partial \theta_s}{\partial r} = 0, \text{ for } r > 1 \quad (9)$$

$$(1-r^2) \frac{\partial \theta_b}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \theta_b}{\partial r}, \text{ for } r < 1, \quad (10)$$

where  $(r, x)$  are the dimensionless variables defined by (5) (since only dimensionless variables will be used below, asterisks will be omitted for convenience). The dimensionless temperature in the ground and the pipe is defined by the formula

$$\theta_{s,b}(r, x) = \frac{T_{s,b} - T_1}{T_0 - T_1}, \quad (11)$$

where the subscripts  $s$  and  $b$  refer to the soil and the brine, respectively.

### 3 BOUNDARY CONDITIONS

Let us formulate the boundary conditions for problem (9), (10). Let the temperature and heat fluxes at the interface between the pipe and the ground be continuous:

$$\theta_s = \theta_b, \text{ for } r=1, \text{ for all } x \quad (12)$$

$$\frac{\partial \theta_b}{\partial r} = \frac{\lambda_s}{\lambda_{br}} \frac{\partial \theta_s}{\partial r}, \text{ for } r=1, \text{ for all } x \quad (13)$$

Moreover,

$$\theta_s \rightarrow 1 \text{ as } r \rightarrow \infty, \text{ for all } x \quad (14)$$

and  $\theta_b$  is continuous at  $r = 0$ .

As (10) is parabolic, only one boundary condition is required for the longitudinal coordinate  $x$ . Therefore, to make the solution of the boundary-value more convenient, we place the origin of coordinates  $(x, y)$  at the point where the brine exits the evaporator of the heat pump (Figure 1). Then formula (11) yields

$$\theta_b = 0, \text{ for } x = 0 \text{ and } r < 1 \quad (15)$$

Thus, the system of equations (9), (10) with the boundary conditions (12)–(15) and the assumptions made above describes the energy exchange among the soil, the vertical heat exchanger, and the heat pump. In such a problem statement, the geothermal system is a single thermodynamic system in which the energy exchange among the soil, the vertical heat exchanger, and the heat pump is coupled processes. This makes it possible to design the optimal GS.

Note that the boundary-value problem depends only on one dimensionless parameter  $\gamma = \lambda_s/\lambda_b$ , which is the ratio of the thermal conductivities of the soil and brine that characterizes the heat exchange between them. This problem obviously has the following two asymptotic solutions:

- 1) if the heat conductivity of the soil is much higher than that of the brine ( $\gamma \gg 1$ ), then the solution in the soil is  $\theta_s = 1$ . This means that the temperature on the surface of the heat exchanger is constant, which provides the best conditions for the extraction of geothermal heat.
- 2) if the heat conductivity of the soil is much lower than that of the brine ( $\gamma \ll 1$ ), then the whole heat exchange process is determined by the thermodynamic properties of the soil. This is the worst conditions for the extraction of geothermal heat.

The fact that the problem includes only one parameter means that all local and integral dimensionless characteristics of this problem or the compound thermodynamic system consisting of soil, heat exchanger, and heat pump are functions of the parameter  $\gamma$  alone. For example, the length  $L_0$  of the vertical heat exchanger needed to generate thermal capacity  $E_0$  is expressed as follows:

$$L_0 = \varphi(\gamma) \cdot \frac{E_0}{\lambda_b \Delta T}, \quad (16)$$

where  $\varphi(\gamma)$  is a function to be determined;  $\Delta T$  is the temperature difference between the inlet and outlet of the evaporator.

The length of a vertical heat exchanger is inversely proportional to the temperature difference between the inlet and outlet of the HP evaporator. Hence, the higher the temperature difference (or, what is the same, the extracted heat) provided by the HP evaporator, the shorter the length of the ground heat exchanger needed to generate the same capacity.

The physical meaning of this phenomenon can easily be explained. Indeed, to obtain the given capacity  $E_0$  by increasing  $\Delta T$  according to (1), it is necessary to reduce  $Q$  or, what is the same, the velocity of the brine flow in the borehole. A decrease of the velocity of the brine improves the heat exchange conditions between ground and borehole.

In the general case, the boundary-value problem has no simple analytic solution; therefore, it will be solved numerically. Since the solution of the problem depends on one parameter, its physical interpretation and possible applications will be quite clear.

#### 4 ANALYSIS OF THE RESULTS

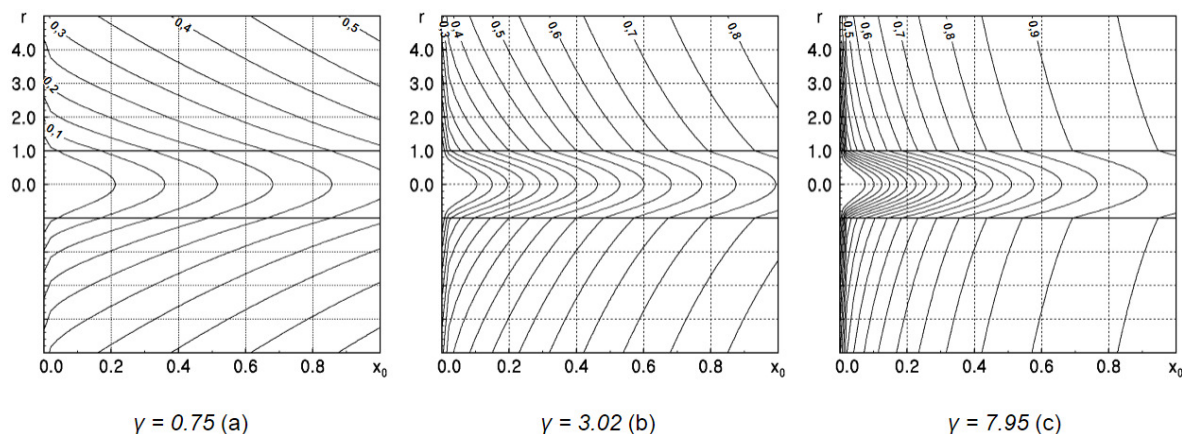
The boundary-value problem (9), (10), (12)–(15) has been numerically solved for various types of soil and brine, which is a mixture of 30% propylene glycol and 70% water with  $\lambda_{br} = 0.44 \text{ W/m K}$  (Ochsner 2008). The types of soil, their thermal conductivities and values of  $\gamma$  are summarized in Table 1. Specific types of soil and brine are used here for illustration and show the range of variation in  $\gamma$  in real conditions. The results are naturally valid for any pair of brine and soil characterized by some value of  $\gamma$ .

**Table 1: The types of grounds, brine, their thermal conductivities and values of**

Brine / soil	30% propylene glycol and 70% water	Dry sand	Dry soil	Sand with 10% water content	Light soil	Sand with 20% water content	Soil with 10% water content	Soil with 20% water content	Argil	Granite
—	0.44	0.33	0.4	0.97	1.16	1.33	1.75	2.1	2.33	3.5
—	1	0.75	0.91	2.20	2.64	3.02	3.98	4.77	5.30	7.95

Values of thermal conductivities are obtained from (Khrgian 1969).

Figure 3a-c shows the calculated distribution of the dimensionless temperature over the soil-heat exchanger system for (dry sand) - Figure 3(a), (wet sand with 20% water content) - Figure 3(b), and (granite) - Figure 3(c). Curves 1, 2, 3 correspond to , , , respectively. The abscissa axis indicates the dimensionless length of the heat exchanger, and the ordinate axis indicates the radial variable. The region represents the brine in the pipe of the heat exchanger, and the region represents the soil.



**Figure 3 (a-c): Distribution of the dimensionless temperature over the soil-heat exchanger system**

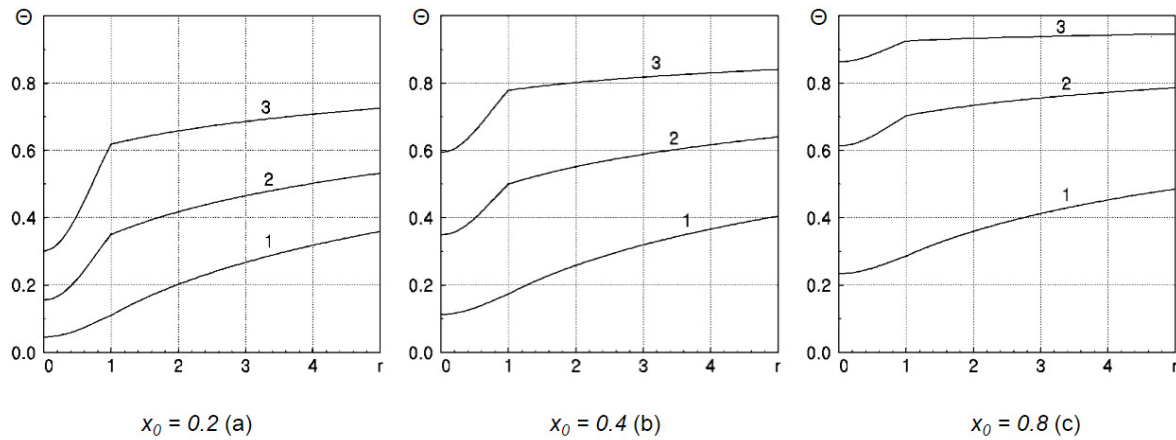
The distribution of dimensionless temperature is represented by isotherms with values indicated in the upper portion of the figure. The results demonstrate strong dependence of temperature on the length of the heat exchanger and on the radial variable, the temperature distribution becoming more nonuniform with increase in and less nonuniform with increase in .

These dependences are better seen in Figure 4a-c which shows the distribution of dimensionless temperature over sections

respectively, for three values of . It can be seen that with increase in the length , the temperatures distribution becomes more uniform both inside the heat exchanger and in the ground, but the temperature remains strongly dependent on .

In case 1 (dry sand), the temperature on the surface of the heat exchanger increases from at to at . For granite , the temperature on the wall of the heat exchanger increases from at to at . The physical meaning of these results is obvious: an increase in for a certain brine leads to an increase in the thermal conductivity of the soil, which improves the heat exchange between the soil and the brine. Noteworthy is the strong dependence of the dimensionless

temperature of the brine on the length of the heat exchanger and the radial variable . This circumstance is rather important for the calculation of the capacity of the heat exchanger.



**Figure 4 (a-c): Distributions of dimensionless temperature over sections**

Let us define the capacity of the energy well (or its debit, by analogy with oil and gas wells) as follows:

$$E = c_p \rho \int_S [T_b(r, x_0) - T_b(r, x_0 = 0)] V(r) ds, \quad (17)$$

where is the temperature of the brine at the inlet of the evaporator; is the temperature of the brine at the outlet of the evaporator; is the longitudinal distribution; is the cross-sectional area of pipe of the heat exchanger. Formula (17) is the difference of the enthalpies of the brine at the inlet and outlet of the HP evaporator. It is obvious that if the temperatures at the inlet and outlet of the evaporator are constant, formula (17) coincides with formula (1). These results indicate that the debit of the energy well can be correctly determined if the dependence of the brine temperature on both length and radius is taken into account.

Let us define the dimensionless debit of the energy well as

$$E(x_0) = \frac{E}{c_p \rho \Delta T \cdot Q}; \quad (18)$$

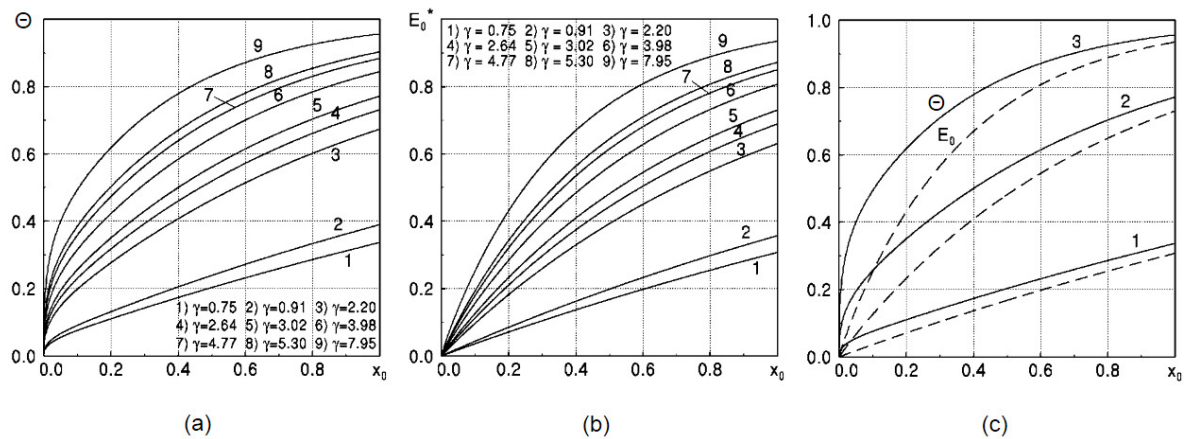
Physically, expression (18) is the ratio of the stationary debit of an energy well of length to the debit of a well with . It is obvious that . It is easy to show that

$$E(x_0) = 4 \int_0^1 \theta_b(r, x_0) (1 - r^2) r dr \quad (19)$$

Figure 5a-c shows the debit of the energy well as a function of its length for various types of soil. Figure 5a represents the debit of the well when the temperature of the brine is independent of and is equal to the temperature of the wall at . In this case, formula (19) gives

$$E(x_0) = \theta_s(r = 1, x_0) \quad (20)$$





**Figure 5: (a) distribution of dimensionless temperature on the collector's wall  
 (b) debit of the energy well as a function of its length  
 (c) debit of the energy well for various type of soil  $\gamma$**

Figure 5b represents the case where the temperature of the brine depends on the radius. It is natural that this dependence reduces the debit of the well compared with the case of radius-independent temperature of the brine. The curves in Figure 5a,b are almost straight when  $\gamma = 0$  and display nonlinear behavior when  $\gamma > 0$ . Physically, the case of  $\gamma = 0$  means that the limiting stage of heat exchange in the well–soil system is heat transfer in the ground; therefore, the temperature profiles are more uniform compared with the temperature profiles for  $\gamma > 0$  where the limiting stage is heat transfer in the brine. Both figures demonstrate that the temperature at the outlet of the heat exchanger and its debit are always lower than those for an undisturbed medium with  $\gamma = 0$  and asymptotically tend to the undisturbed level as  $x_0 \rightarrow \infty$ . The physical meaning is obvious: the soil–heat exchanger–heat pump system is in a stationary state attained after long operation period starting from the undisturbed state. Since the debit of the well is nonlinear in  $x_0$  when  $\gamma > 0$ , it follows from Figure 5b that a required debit can be achieved with several wells. For example, when  $E_0^* = 0.5$ , then can be obtained with two wells with  $x_0 = 0.5$  (Figure 5b). One well could provide such a capacity if its length is infinite. However, if we have one well of length  $x_0 = 1.0$  instead of two wells of length  $x_0 = 0.5$ , then  $E_0^* = 0.5$ . These results can be used to design a heat exchanger of optimal length that would provide the maximum capacity of the well. Figure 5c shows the debit of the wells as a function of their length for dry sand (curves 1), sand with 20% water content (curves 2), and granite (curves 3). The solid curves correspond to  $\gamma = 0$ ; the dashed curves represent the debit of the wells presented in Figure 5b. As one would expect, the difference between them decreases as  $x_0 \rightarrow \infty$ . Figure 6a shows the absolute error in the determination of the debit of the wells based on the temperature of the wall. For  $\gamma = 0$ , this difference is almost constant; when  $\gamma > 0$ , the error has a maximum of  $\Delta E_0^* = 0.1$  at  $x_0 = 0.5$  and  $\Delta E_0^* = 0.2$ . Figure 6b shows the length  $x_0^*$  of the heat exchanger at which the error of the debit determination  $\Delta E_0^*$  is maximum versus the parameter  $\gamma$ .

These results demonstrate that the non-uniformity of the longitudinal and radial distribution of the brine temperature has a strong effect on the capacity of the energy well. This circumstance necessitates a revision of the existing engineering methods for determining the capacity of an energy well based on a thermal response test (TRT) which assumes constancy of heat fluxes along the heat exchanger. Figure 7a shows the average capacity per unit length  $E_0^*/x_0$  of the energy well as a function of the length of the heat exchanger and the parameter  $\gamma$ . It can be seen that for  $\gamma = 0$  (poorly conductive media) the thermal capacity per unit length of the heat exchanger is weakly dependent on its length  $x_0$ , while for  $\gamma > 0$ , this dependence is essentially nonlinear, especially for small  $x_0$ . With increase in  $x_0$ , the average capacity of the well tends to zero. The physical meaning of this phenomenon is

illustrated by Figure 7b, which shows how the local dimensionless heat fluxes from the ground to the heat exchanger along its length depends on the parameter  $\gamma$ . For  $\gamma = 1$ , strong thermal nonuniformity forms within the initial section of the heat exchanger and heat fluxes are maximum here. With increase in  $\gamma$ , the local heat fluxes reduce. For  $\gamma = 10$  (granite), the fluxes decrease by more than an order of magnitude over length  $x_0$ . This means that in well-conductive soils, required capacity can be generated with several short wells whose total length can be much less than the length of a single well of the same capacity.

maximum on parameter  $\gamma$

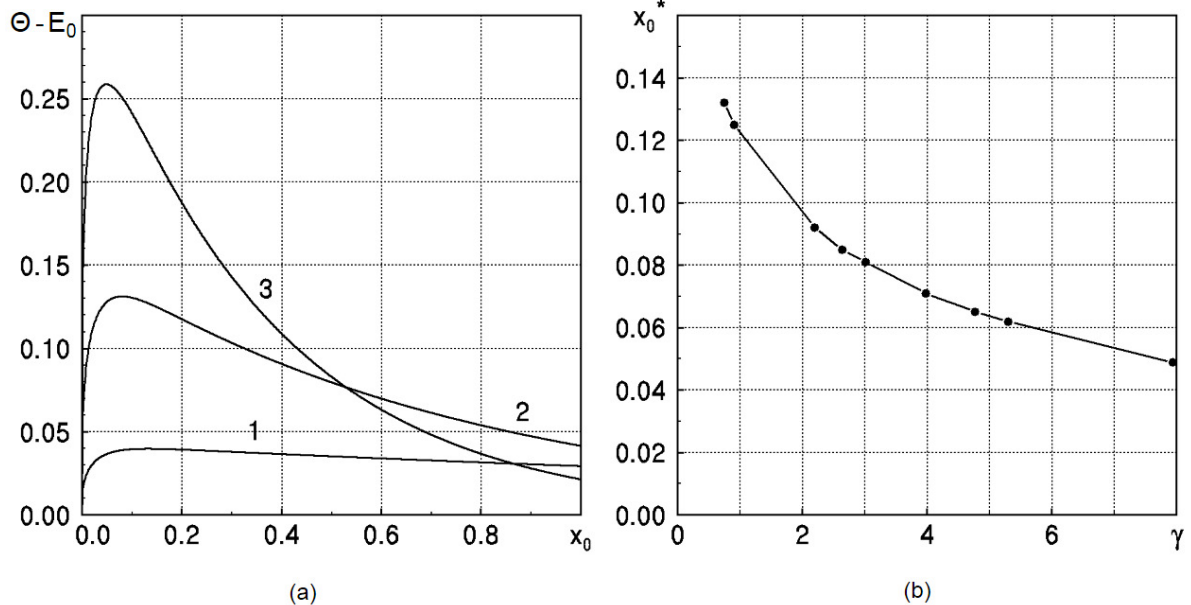


Figure 6: (a) absolute error in the determination of the debit of the wells based on the temperature of the wall  
(b) dependence of error

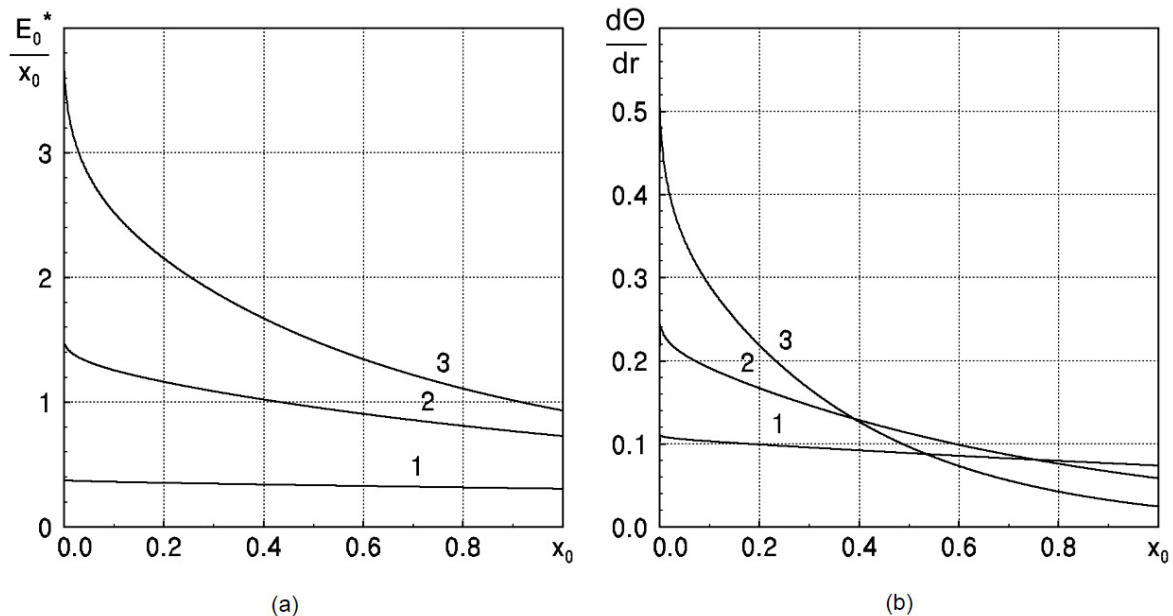
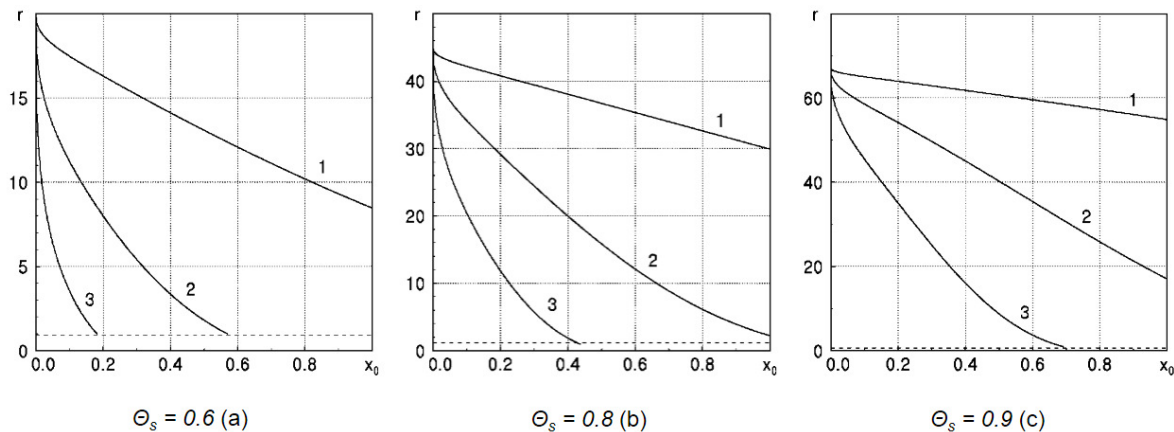


Figure 7: (a) average capacity per unit length of the energy well  
(b) local heat fluxes from the ground to the heat exchanger along its length

When several wells are used, it is very important to consider their mutual influence. To this end, it is necessary to determine the size of the disturbed region of the temperature field in

the ground. Figure 8a-c shows, for three values of  $\Theta_s$ , the regions of disturbed temperature fields bounded by the isotherms  $\Theta_s = 0.6$ ,  $\Theta_s = 0.8$ , and  $\Theta_s = 0.9$ , respectively.



**Figure 8 (a-c): size of the regions of disturbed temperature fields by the three isotherms**

The dashed line represents the wall of the heat exchanger. The size of the disturbed region increases with a decrease in the parameter  $\Theta_s$ , but its linear size on the surface remains almost the same (it is equal to  $x_0 \approx 0.6$  (or to  $x_0 \approx 0.6$  if in dimensional form) for the isotherms  $\Theta_s = 0.6, 0.8, 0.9$ ). For polyethylene pipes that are usually used in heat exchangers, we have  $\lambda_{pe} \approx 0.4$  W/mK. Then, the maximum linear size of the disturbed region on the surface is on the order of 2 m. Hence, the distance between wells for different soils should be no shorter than 4 m, which is in good agreement with the engineering recommendations (Ochsner 2008).

## CONCLUSIONS

The mathematical model of a strongly non-equilibrium thermodynamic systems G - BHE - GSHP was elaborated to increase understanding of thermodynamic interaction of all elements of a GSHP system. Based on this model the stationary problem of this system operation has been solved. In particular, it is shown that:

- the stationary energy exchange processes in the system G - U-tube BHE - HP are characterized by a unique dimensionless parameter - the ratio of the thermal conductivity of the ground and brine ( $\Theta_s$ );
- the strong dependence of temperature on the length of the heat exchanger and on the radial variable, the temperature distribution becoming more non-uniform with increase in  $\lambda_{pe}$  and less non-uniform with increase of well length;
- with increase in the length  $L$ , the temperatures distribution becomes more uniform both inside the heat exchanger and in the ground, while the temperature remains strongly dependent on  $\lambda_{pe}$ ;
- the capacity of the energy well can be correctly determined if the dependence of the brine temperature on both length and radius is taken into account;
- the non-uniformity of the longitudinal and radial distribution of the brine temperature has a strong effect on the capacity of the well;
- in the well-conductive soils the required capacity can be generated with several short wells whose total length can be much less than the length of a single well of the same capacity.

Moreover, a definition of a debit of energy well was introduced and the limiting stages of heat exchange in the well-soil system were identified.

Finally, the size of the disturbed region of the temperature field in the ground was determined. The numerical calculations showed the good agreement with the engineering recommendations.

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