A NOVEL NUMERICAL MODEL FOR THE THERMAL RESPONSE OF BOREHOLE HEAT EXCHANGER FIELDS

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Abstract: The design of a borehole field should be based on a long-term simulation of its thermal response for specified energy loads. This response can be calculated from a pre-calculated dimensionless function, the g-function. This paper is focused on a new approach to the calculation of g-functions; in particular with a precise representation of the boundary conditions at the borehole wall. First, the almost constant temperature along the borehole heat exchanger requires a boundary condition of essentially isothermal boreholes along the depth. In a common case the borehole heat exchangers are connected in parallel, thus all boreholes should have the same temperature. Also, the total heat flow to the borehole field should be constant over time. A numerical model in which the boreholes are filled with a highly conductive material has been built, reproducing the isothermal condition. By thermally interconnecting the boreholes, the equal temperature condition is satisfied. The g-functions of some simple borehole field configurations are presented in this paper. The results show, in general, a good agreement with the existing solutions for a similar boundary condition.

Key Words: borehole, heat exchanger, g-function, long-term performance, uniform borehole temperature.

1 BACKGROUND

During the last years, Ground Source Heat Pump (GSHP) systems have become increasingly popular among the renewable technologies due to their high efficiency and their contribution to mitigation of CO_2 emissions.

The most common GSHP systems in Sweden are those coupled to vertical borehole heat exchangers (BHEs), with a total of about 425,000 small (family houses) systems and 400 large Borehole Thermal Energy Storage (BTES) installations. Some statistics presented in (Andersson and Bjelm 2013) state that more than 150 units with more than 6 boreholes were reported in 2011. While the growth rate of new small systems has slowed down, larger GSHP installations are nowadays starting to take a significant role in Swedish energy statistics.

The design of large BHE fields should account for extrinsic factors such as a realistic assessment of the heating and cooling demand of the building, the temperature limitations of the secondary fluid, and the available drilling area. Moreover, intrinsic factors such as the thermal properties of the ground and the possible presence of groundwater flow should be evaluated. Taking all these factors into account, the design results in a borehole (BH) field with a particular arrangement comprising a certain number of boreholes of prescribed lengths. Well-known commercial software programs such as Energy Earth Designer (EED) (Hellström and Sanner 1997) and GLHEPRO (Spitler 2000) are used to design BHEs and

simulate the thermal response of the ground for many relatively simple borehole field geometries.

The above mentioned software programs contain a library of pre-calculated response factors, the so called g-functions. The g-functions implemented in these software programs were obtained from one solution of the so called Superposition Borehole Model (SBM) (Eskilson 1986), with a uniform temperature condition at the BH wall.

The concept of the g-function, first presented in (Eskilson 1986) and (Eskilson 1987), included the use of dimensionless variables, r_b/H and B/H, representing the aspect ratios to the active BH length (H) of the BH radius (r_b) and BH spacing (B), respectively. The g-function depends also on a non-dimensional time t/t^* where t^* is a characteristic time defined as the Fourier number with H as the characteristic length and α as the thermal diffusivity of the ground, that is $Fo_H = \alpha t/H^2$. The upper inactive part of the BHs, D, which could be the groundwater level or an insulated casing was dismissed as a negligible parameter in the g-function definition.

The g-function is a response factor for a specific geometry that relates the change of temperature at the BH wall,$T(t)$, over time with a constant average heat flow per unit BH length, q, imposed from time $t = 0$. Equation 1 expresses the link between the g-function, the heat load, and the temperature, for a BH field buried in a ground with a thermal conductivity, k. T_0 is the undisturbed ground temperature.

$$T(t) - T_0 = \frac{q}{2\pi k} * g\left(\frac{t}{t^*}, \frac{r}{H}, \frac{B}{H}\right) \qquad (1)$$

The drawback of numerical methods is a long computing time for the g-function generation, especially for large borehole fields. This is the reason for implementing a library of g-functions in commercial software. Although the library contains hundreds of g-functions for different multiple BH field configurations (most patterns are linear, square or rectangular), the users cannot always choose their own special configuration. In case the pattern is found in the library, sometimes the aspect ratios of the BH field are different from those specified in the library for that prescribed pattern. Then, the software interpolates a g-function from a set of known g-functions contained in the library, which might lead to some loss of accuracy (Yang et al. 2010). The logarithmic interpolation may overestimate the g-function values by a few percent, 4 % or less. (Malayappan and Spitler 2013).

As indicated in Equation 1, the main use of the g-function is in the simulation of the long-term thermal response of a BH field exposed to variable heat loads. It should be said that a straight-forward calculation of this response for a multi-BH field with variable loading conditions is normally not within the capability of the BH field designer. To simulate such thermal response, a temporal superposition of variable loads is applied to the g-function, as mathematically expressed in Equation 2. The variable load is split into n piecewise constant load steps q_i, starting at t = 0 with q_1 lasting to t_1 where the next step q_2 starts and so on until the last step q_n ending at t_n. It should be noted that the geometrical aspect ratios have been omitted in the argument of the g-function in Equation 2.

$$T(t) - T_0 = \frac{1}{2\pi k} * \left(q_1 * g\left(\frac{t}{t^*},,\right) - q_n * g\left(\frac{(t-t_n)}{t^*},,\right) + \sum_{i=1}^{n-1}(q_{i+1} - q_i) * g\left(\frac{(t-t_i)}{t^*},,\right)\right) \quad (2)$$

The calculation of g-functions for multiple BH fields is a demanding and time consuming task. A variety of methods are in use, e.g. analytical, (semi)analytical and numerical based on Finite Difference (FDMs) and Finite Element Methods (FEMs).

An analytical approach is the Finite Line Source (FLS) model, which allows the generation of the g-function for any borehole configuration. Here, the boundary condition is restricted to all boreholes having the same heat flux at the borehole wall. This analytical solution was first introduced by Eskilson and modified later by (Zeng et al. 2002) and (Lamarche and Beauchamp 2007).

A crucial point when generating a g-function is the boundary conditions at the BH wall, which requires three simultaneous conditions to be fulfilled. All BHEs are in general hydraulically connected in parallel and the temperature drop in the passage of the BHE is for physical reasons so small that the active part of the BH may be considered as isothermal, at least in the often used U-pipe BHEs. This leads to the conclusion that the walls of all boreholes should have a quasi-uniform temperature, but changing over time as the ground responds to the heat load. The heat load should be constant over time, meaning that the heat distribution between the BHs also changes with time as the BHs begin to thermally interfere with each other and with the surrounding ground.

The g-functions generated for identical BH configurations but using the uniform temperature (and constant total heat flow applied to the BH field) or the constant heat flux boundary condition show discrepancies that become more significant for large BH fields and large time scales. The advantage of the FLS approach is its flexibility to generate the g-function of any BHE configuration and the fast computation with an acceptable accuracy (Fossa 2011a and 2011b). Particularly for analytical methods the uniform temperature condition is difficult to manage and often constant heat flux at the BH wall is set as boundary condition instead. Recent ways to handle the boundary condition limitation of the analytical methods are presented in (Cimmino et al. 2013), (Lazarotto 2014) and (Cimmino and Bernier 2014). All these are based on the FLS analytical approach, which normally assumes constant heat flux. The latter fulfills the uniform temperature condition at the borehole wall by sectioning the boreholes along the depth (Cimmino and Bernier 2014) and solving the analytical equation for each section. (Lazarotto 2014) connects BHs with each other being able to account for the temporal variation of the heat load between BHs while keeping a constant total load.

In BH fields with both heat extraction and injection, repeated on a periodic, usually annual basis, only the net heat load will play a role in the long-term temperature response. In nearly balanced BH fields, the major part of the temperature response will thus be confined to the load variations within the yearly period and the corresponding part of the g-function, limited in a practical case to ln (9FoH) < about -3. This is where the differences between the discussed g-functions are small. The net load is small compared to the annual load turnover in a nearly balanced case and also when the long-term differences in the g-functions are significant, the impact on the total temperature response will be small, (Monzó et al. 2013b).

This paper presents a novel numerical model in which special attention is given to imposing the isothermal condition at the BH wall, by filling the BH with a highly conductive material (HCM). The model is a continuation of previous work in which constant heat flux boundary condition was set at the BH wall (Monzó et al. 2013a and 2013b). Moreover, the BHs are thermally connected to represent an equal temperature condition as a constant total heat flow to the borehole field is applied. The g-functions are calculated from this model for some simple borehole fields, a single BH and a 2x3 BHs configuration, and compared with some reference solutions.

This model is also being used to simulate the thermal response of real variable loads in existing GSHP installations (Monzó et al. 2014). The model is flexible and allows imposing any boundary condition at the BH wall. The methodology explained in this paper has also been applied to study the thermal response of large multi-BH fields with a quasi-uniform temperature gradient between top and bottom of the BH in (Peréz 2013) and (Penttilä et al. 2014).

2 DESCRIPTION OF THE NUMERICAL MODEL

This section describes a FEM in which the transient heat conduction problem in BHE fields is studied from the BH wall out into the surrounding ground, a continuation of previous studies that were focused on the g-function generation by assuming constant heat flux at the BH wall (Monzó et al. 2013a). The present model uses a correct boundary condition by imposing a uniform temperature at the BH wall. Since the BHs are hydraulically connected in parallel, they should be thermally connected; consequently the boundary condition at the BH wall is set to represent a uniform temperature along the BH length in all BHs.

2.1 g-function generation for a single borehole

First, the model is shown for the simplest case, a single BH in a 2D-axisymmetric space dimension with the BH aligned with the vertical axis. The active length of the BH is 100 m with a BH radius of 0.05m. A prescribed groundwater level of 4 m is considered as inactive depth, D. The surrounding ground is represented by a big square with sides of 1,000 m. It should be noted that the dimensions of the surrounding ground is not required to be as big as set in our model. However, this model is being used to test other parameters and the authors prefer to keep this size not to compromise any results.

Table 1 shows the aspect ratios for this single BH configuration. The thermal properties of the ground are defined in terms of thermal conductivity, 3.5 W/mK, specific heat at constant temperature, 850 J/kgK and density, 2,600 kg/m^3.

Table 1 : Geometrical aspect ratios of the single borehole model

Geometrical Aspect Ratio	Value
r_b/H	0.00050
D/H	0.040

The initial condition is set to zero temperature in the whole domain and at all the boundaries. The boundary conditions at all domain boundaries, except at the BH wall, are also set to zero temperature. This zero condition represents the undisturbed ground temperature.

The heat transfer problem inside the BH is modeled by considering a fictitious HCM, which is embedded inside the BH. The thermal conductivity of the HCM is chosen so that the temperature is almost uniform along the BH length to give a fair representation of the thermal gradient when the BH is in operation. After several iterations, the HCM thermal conductivity has been set to 10^{10} W/mK.

As regards the mesh, it consists of free triangular elements. Their size varies with smaller elements close to the BH and larger further out with around 25,000 elements in total. A transient study is set for a sufficiently long time in order to generate a g-function for values up to $\ln(9Fo_H)$ around 2. The computing time is about 20 minutes on a computer with 32 GB RAM and an Intel© Xeon© processor CPU E5-2620 at 2.00 GHz (2 processors).

2.2 g-function generation for a 3x2 BHs configuration

The model becomes more complex when the number of BHs increases. The previous study is here expanded to multi-BH geometry, 3x2 BHs in a symmetric configuration. This model is built in a 3D space dimension. In this case, the BH dimensions are the same as in the single BH case with a BH spacing of 5 m. Figure 1 shows a representation of the modeled 3D domain. The total modeled domain is half the size of the BH field (heat transfer symmetry is

applied to reduce the BH field by half), and the surrounding is 200x200 m in the horizontal plane with 300 m depth. Table 2 shows the aspect ratios for this BH configuration.

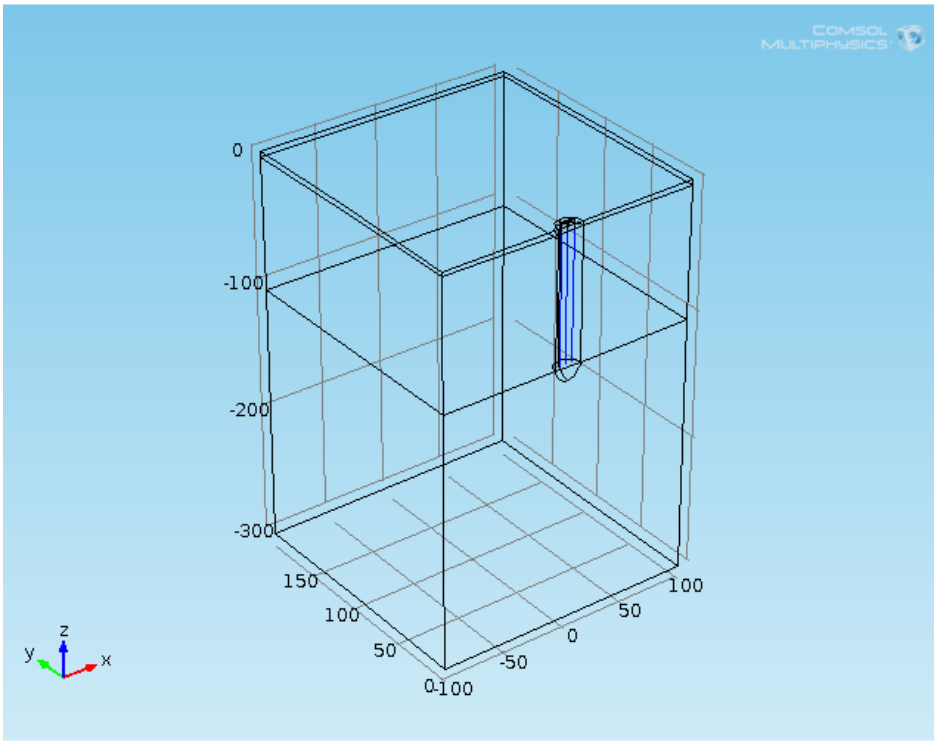


Figure 1 : Computational domain of 3x2 BHs configuration

Table 2 : Geometrical aspect ratios of the 3x2 BHs configuration

Geometrical Aspect Ratio	Value
r_b/H	0.00050
B/H	0.050
D/H	0.040

The same considerations as for the single BH are taken with regards to the boundary conditions and material properties. However, in this case, the uniform temperature condition at the BH wall in all the BHs has to be satisfied, i.e. the BHs need to be thermally connected when the BH field comprises more than one BH. For that purpose, the BHs are physically connected with a bar of the same HCM material. This bar is connected to the HCM-filled BHs by cones of the same HCM. Relevant surfaces of the rod and the cones are insulated. The heat load is applied on the top surface of the bar and it is naturally distributed through the HCM material as governed by the thermal response of the ground, keeping a uniform temperature at all the BHs walls. Figure 2 shows a sketch of the connection between the BHs for the modeled geometry.

The HCM bar is elevated somewhat to stay off from the ground surface. The total length of the cone should be the sum of D, the upper insulated part of the borehole, plus a certain elevation, E, to ensure that the cone is embedded in the bar. Thus, the top side of the cone is embedded in the bar, whereas the bottom part of the cone is in contact with the top surface of the BH filling. The thermal conductivity of the HCM is again set to 10^{10} W/mK, and it applies to the bar, cones and inner domain of the BHs. No heat transfer occurs through the

lateral surface of the cone. Table 3 shows the geometrical characteristics of the bar and cone.

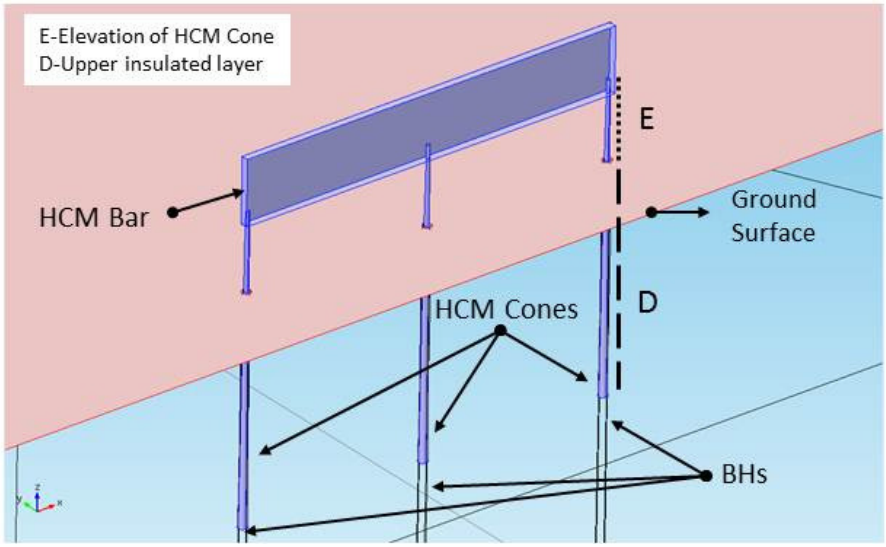


Figure 2 : Connection of the BHs to HCM bar. Half geometry of 3x2 BHs configuration

Table 3 : Geometrical characteristics of the bar and cones used to connect the boreholes

Element	Dimension	Size
Bar	Width [m]	10.20
	Depth [m]	0.20
	Height [m]	1
Cone	Bottom radius [m]	0.05
	Height [m]	5.20
	Top size [deg]	Semi-angle 0.5

Regarding the mesh, a free triangular mesh is defined on the fictitious layer at the upper part, D. This mesh presents small elements in the region close to the borehole which increase as the distance from the borehole field increases. Then, the triangular elements are swept along the BH length, creating a mesh of radial elements. The upper inactive part and the downward region of the BH field are meshed with tetrahedral elements. The bar, the cones and the inner part of the BHs are meshed in a similar way. The mesh consists of 250,000 elements. The model is set as a transient study for long term simulations, i.e. reaching values of $\ln(9Fo_H)$ equal to around 2 (about 170 years for the values used in this paper). The computing time is around 150 minutes on a computer with the aforementioned characteristics.

3 GENERATION OF g-functions AND DISCUSSION

In this section, the g-function generated from our numerical model is compared with the g-functions obtained using analytical FLS solutions (constant heat flux at the borehole wall) as well as those generated using the SBM solution with uniform temperature condition.
In Figure 3 and Figure 4, the results referred to our numerical model are identified as “FEM-Uniform T”.

3.1 g-function for a single borehole

Figure 3 shows the g-function for a single borehole.

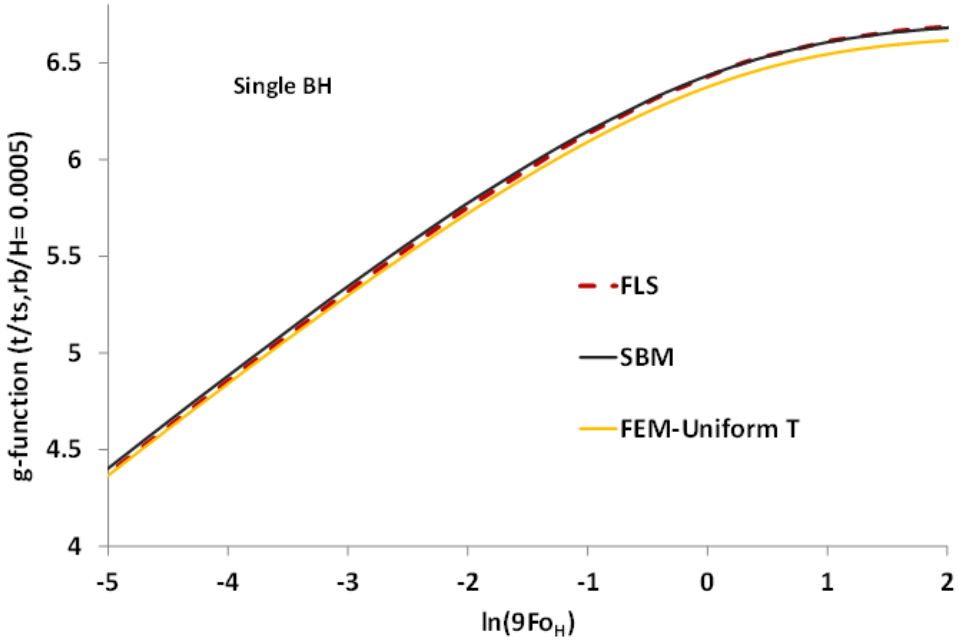


Figure 3 : g-function for a single borehole

In the asymptotic part of the curve, at about $\ln(9Fo_H)=2$ or 170 years, for the given parameters in this paper, the differences between "SBM" and "FEM-Uniform T" solution are around 0.07 units. At a short time, $\ln(9Fo_H)=-4.5$ or 90 days approx., the difference becomes around 0.04 units. For the given value of the thermal conductivity and a constant heat flux injection of 20 W/m, the temperature differences are about 0.035 and 0.06 K for 90 days and 170 years, respectively. The "FLS" solution comes out between the "SBM" and "FEM-Uniform T" g-functions. In comparison with "SBM", "FLS" deviates around 0.02 units at short time. For the single borehole problem, no major differences are found when changing the boundary condition. All solutions are in general the same since all of them consider the finite length of the borehole. The FLS solution, however, is different at shorter times, given that it is based on a line source at the BH center and not a source at the borehole wall. All curves meet at about $(\ln(9Fo_H)=-11.39)$ and even with a smaller error at $(\ln(9Fo_H)=-10.01)$. This time period is not included in Figure 3.

This situation will obviously become different for multiple borehole problems when borehole interaction affects the validity of the constant heat flux boundary condition.

3.2 g-function generation for a 3x2 BHs configuration

Figure 4 shows the g-function for the 3x2 BHs configuration. In "FLS" and "FEM-Uniform T" solutions are calculated with a D/H=0.04 m. However, the "SBM" solution, as mentioned above, does not clearly specify the value assumed for D.

The g-function calculated from the three approaches present small deviations for times when $\ln(9Fo_H)<-2$, that is about 3 years (for the ground thermal properties described in section 2). In general, the "FEM-Uniform T" shows a better agreement with SBM, and presents lower values. The slight deviation of "FEM-Uniform T" from "SBM" is about 0.16 units at short times $(\ln(9Fo_H)=-4.5)$. The difference increases up to 0.28 units in the asymptotic part $(\ln(9Fo_H)=2)$. In terms of temperature differences, for a thermal conductivity of 3.5 W/mK and a constant heat injection of 20 W/m, the "FEM- Uniform T" deviates from "SBM" about 0.14 and 0.26 K for the aforementioned interval of times, short and long term respectively.

The FLS is very close to the SBM in the beginning, slowly reaching a deviation from SBM of 0.06 units at ln(9FoH)=-2. Thereafter the deviation increases at a faster rate reaching 0.60 units near the asymptotic end of the curve whereas the "FLS" solution comes out significantly higher (the deviation of the FLS solution for larger times is explained in many previous studies, see section 1).

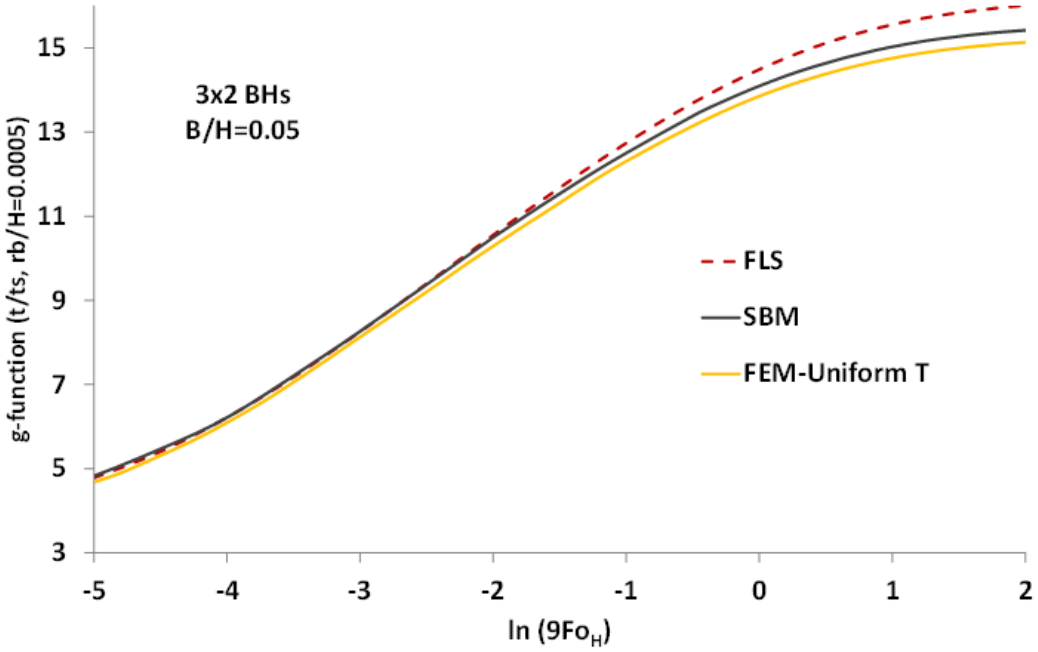


Figure 4 : g-function for a 3x2 BHs configuration

In general, In comparison with "SBM", the g-functions from "FEM-Uniform T" present a good agreement with slightly lower values. The g-function obtained from "FEM-Uniform T" presents small deviations either for the single BH or for the 3x2 BHs configuration. These differences might be related to the different value of D considered in "SBM" and "FEM-Uniform T". There might also be an association with some adjustable features in the solver of our numerical model. Further work will be aimed at studying these differences. Using SBM as an absolute reference could also be questioned.

4 CONCLUSION

A novel numerical model to simulate the thermal response of borehole fields is described in this paper. This model pays special attention to the boundary condition defined at the BH wall. A fictitious Highly Conductive Material (HCM) is used to simulate a uniform temperature condition at the borehole wall. The HCM is extended up to a bar to thermally connect all the boreholes within the borehole field. The flexibility of this model allows the study of the thermal response of any borehole field configuration. This model also allows imposing a variety of temperature conditions at the borehole wall.

This new numerical approach is, in this paper, used to generate g-functions for a single borehole and for a 3x2 boreholes configuration, named "FEM-Uniform T" solutions. The results are compared with the solution from, SBM (uniform temperature) and FLS solutions. In general, the "FEM-Uniform T" g-functions present good agreement with the "SBM" solution, having slightly lower values. The deviation becomes higher in the asymptotic part of the curve. In a single borehole configuration the deviation varies from 0.04 (0.035K) units for

short time and 0.07 (0.06K) units in the asymptotic part of the curve. In the case of a 3x2 boreholes configuration, the deviation differ 0.16 (0.14K) units at short times and 0.30 (0.26K) units in the asymptotic part of the curve.

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