A NEW CONTROL APPROACH FOR HEAT PUMP HEATING SYSTEMS

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ABSTRACT

Two concepts of a new control approach for heat pump heating systems are presented. Instead of conventionally controlling either the supply or the return flow temperature in function of outdoor temperature, the new approach directly delivers in adequate portions the heating energy required into the house utilizing a pulse width modulation. The first simpler concept mainly uses the steady-state energy demand of the house, whereas the more sophisticated second concept additionally uses the thermal dynamics of the house. This paper focuses especially on the second concept. It is shown that thermal dynamics of the house, the weather forecast, the power tariff structure, and power cut-off times may be taken into account to optimize both the comfort and an energy or cost efficient operation of the heat pump. Simulation results show that a good comfort level can be achieved with a reduction of power costs of 10% compared to the operation with a commercial relay-type controller. This control concept was implemented in a production-type controller that could be tested successfully as a prototype controller in a single-family house.

1. INTRODUCTION

In conventionally controlled heat pump heating systems equipped with heat pumps operating with single-speed compressors, either the supply or the return flow temperature is controlled by a relay-type controller. The set-point of the controller is chosen as a function of the outdoor temperature along the so-called heating characteristic curve, which characterizes the steady-state heating energy consumption of the building for a certain indoor reference temperature. Restrictions, such as low-tariff periods or cut-off periods determined by electric power providers cannot be integrated for optimization purposes, nor can this simple controller take into account the weather forecast. Additionally, in well-insulated single-family houses with low-temperature heating systems, the heating characteristic curve is rather low and flat, such that the hysteresis band of the controller may cause the heat pump not to switch on above a certain outdoor temperature.

Within the scope of the research project called “Pulse-width modulation for heat pump heating systems” funded by the Swiss Federal Office of Energy, a new control approach has been developed. This approach does not aim at controlling the temperature of either the supply or the return flow. Rather, it quantifies optimal heating energy portions to be delivered into the house, optimally distributed throughout the day with respect to the weather forecast, the most efficient operation of the heat pump, low-tariff periods, and power cut-off periods, while still guaranteeing the desired indoor temperature.

Two concepts of this new control approach are considered. In the simpler concept, only the steady-state heating characteristic curve and the mean value predicted for the outdoor temperature are used to determine the distribution of the heating energy "portions". The user must predefine a preferred size of these "portions". This concept is introduced in section 2 of the paper. In a more sophisticated concept, the duration of these portions is optimized with respect to the most energy-efficient operation of the heat pump. Instead of a steady-state heating characteristic curve, a mathematical model for the dynamic thermal behavior of the house and a predicted transient for the outdoor temperature are utilized in a controller algorithm based on the model predictive control (MPC) approach to optimize the distribution of the heating energy portions. This concept is described in section 3 of the paper. It intensively utilizes mathematical tools from the control theory, such as LQ optimization (LQ: linear model for the system to be optimized, quadratic objective function for the optimization) which is summarized in section 3.3. For a detail introduction to MPC and LQ-optimization please refer to [1], [2], and [3].

In section 3.1 a linear model for the thermal dynamics of the system is presented that is used for the LQ-optimization. The method used for the prediction of the outdoor temperature is described in section 3.2. The optimal control achieved by the LQ-optimization must be modified to a binary signal in order to turn on and off the compressor of the heat pump. This is done by a pulse width modulation which is introduced in section 3.4. The simulation results and the field testing results are presented in the sections 3.5 and 3.6.

2. CONTROL CONCEPT BASED ON THE HEATING CHARACTERISTIC CURVE

In this pragmatic control concept every 15 minutes the total heating energy required for 24 hours is calculated based on both the mean value of the outdoor temperature over the last 24 hours and the energy characteristic curve of the house, which shows the steady state heating energy required for 24 hours in function of outdoor temperature. For the distribution of the heating energy required for each tariff period, the user has to predefine a preferred percentage number for distribution. The duration for the heat pump to be turned on within each tariff period is then calculated by using the power characteristic curve of the heat pump, which shows the steady-state heating power of the heat pump in function of outdoor temperature. Utilizing a preferred duration for heating pulses and heating pauses that also have to be predefined by the user, and taking into account the restrictions given by the power tariff structure as well as the power cut-off periods finally determine the number of pulses required for both low and high tariff periods.

The number of heating pulses are calculated every 24 hours in advance for the subsequent 24 hours. However, the duration of the heating pulses is recalculated every 15 minutes based on the actual heating energy required using the actual mean value of the outdoor temperature. This allows to consider sudden changes in the outdoor temperature. At the end of a 24 hours period an energy balance is drawn. At this time the attempt is to exactly deliver the total amount of energy required for the last 24 hours. A possible remainder will be carried over into the next 24 hours period.

This control concept is implemented in a production-type controller and is currently testing in an occupied single family house.

Figure 1 shows for this control concept the simulation results for a typical day with a preferred heating pulse duration set to 60 minutes, a preferred heating pause duration set to 60 minutes, and a preferred percentage number for the heating energy distribution set to 50% for both the low and the high tariff periods. Clearly, for this simulation with no power cut-off periods assumed, at the end of the day the energy balancing has succeeded.

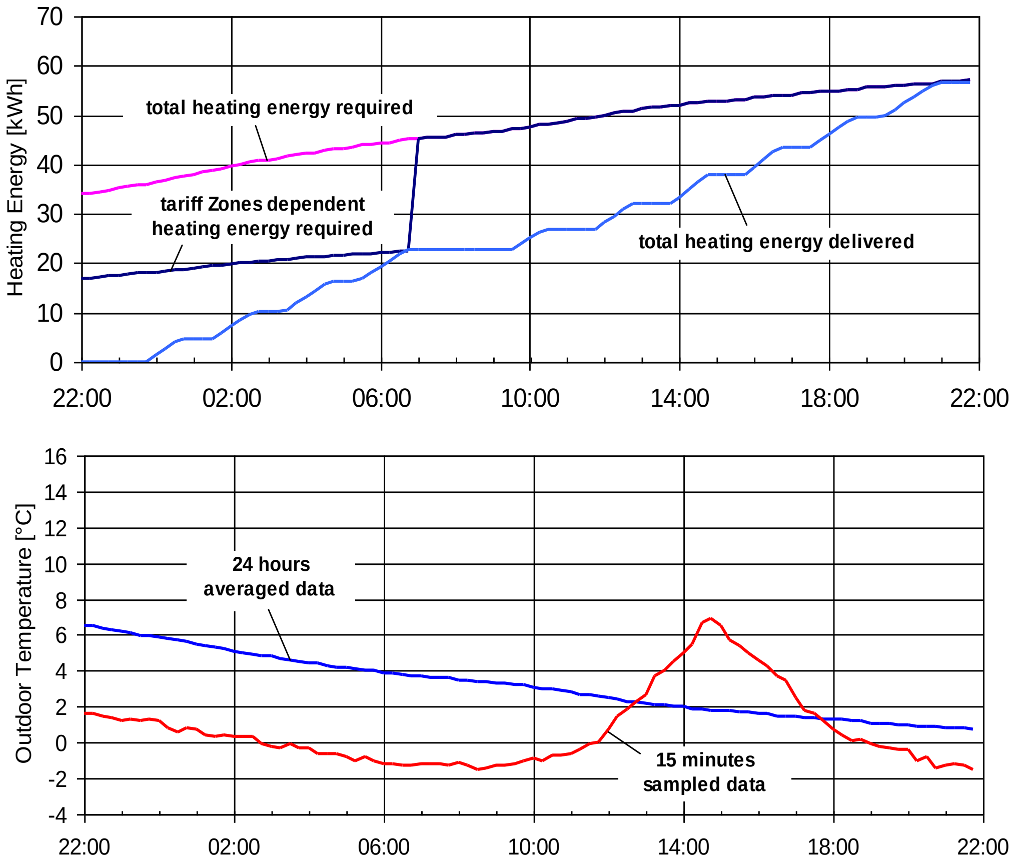

Figure 1. Simulation results for the control concept based on the heating characteristic curve

3. MODEL PREDICTIVE CONTROL

In this more sophisticated control concept, the duration of the heating pulses as well as their distribution over time are optimized with regard to the temperature dynamics of the building, the weather forecast, the power costs (high and low tariff), the power cut-off times, and the most energy-efficient operation of the heat pump. For the realization of this control concept the Model Predictive Control (MPC) approach is chosen. This method is briefly described below. For a detail introduction please refer to [1].

The MPC is a discrete-time control approach. It uses a mathematical model of the system to predict its future behavior based on the current system states, reference signals, and predicted disturbances. It then optimizes the controller output by minimizing a given objective function for a defined prediction horizon. Although the discrete-time control signal is optimized for the whole prediction horizon, only the first discrete value is used for the controller output. At the next sampling time, new values of the system states are obtained, the control horizon is shifted forward by one sampling interval, and the same algorithm is applied again.

For the application presented in this paper, the reference signal is the desired room temperature $T_{ref}(t)$, the predicted disturbance is the outside temperature $\hat{T}_{out}(t)$, and the controller output of the MPC algorithm is either the on/off signal for the heat pump or the heat flow to be delivered by the heat pump $\overset{*}{Q}_h(t)$, depending on the method chosen for the optimization.

To generate the on/off signal for the heat pump, either a direct or an indirect method can be used. In the direct method, the time events for switching the heat pump on and off are the parameters to be directly optimized. In the indirect method used in this paper, in a first step the heat flow required by the building must be optimized. In a second step the on/off signal for the heat pump is generated by utilizing a pulse-width modulator (see Section 3.4).

3.1 Model of the Reference System

The reference system is a single family residential house equipped with a commercial air-to-water heat pump for heating only. The heat pump has a nominal power of 12.2 kW at an outdoor air temperature of 2°C and a supply flow temperature of 35°C. The heating system consists of both a floor heating (70%) and radiators (30%). The house is a modern, heavy construction with well insulated brick walls and a large area of windows oriented towards the Southwest. In the return flow path a water storage tank is mounted between the heat pump and the radiator/floor heating system (Figure 2).

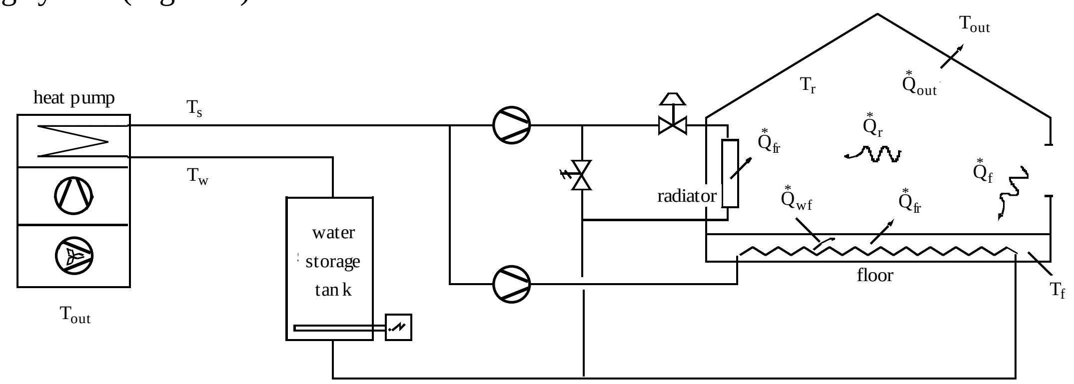

Figure 2. Schematic illustration of the reference system

House model:
To describe the thermal behavior of the house—including the floor heating system, the radiators, and the water storage tank—a third-order, semi-physical model is used. The total amount of water in the radiators, the floor heating system, the tubes, and the water storage tank together are regarded as one heat storage entity as described by equation (1). The second heat storage entity is assumed to consist of the whole mass of the floors and of the radiator walls. The corresponding dynamics are described by equation (2). The house itself is modeled as a single-space building with only one node described by equation (3). The return flow temperature $T_w(t)$, the floor temperature $T_f(t)$, and the room temperature $T_r(t)$ are the corresponding state variables of this model.

$$\rho_w \cdot V_w \cdot c_w \cdot \dot{T}_w(t) = \overset{*}{Q}_h(t) - \overset{*}{Q}_{wf}(t) \quad (1)$$

$$m_f \cdot c_f \cdot \dot{T}_f(t) = \overset{*}{Q}_f(t) + \overset{*}{Q}_{wf}(t) - \overset{*}{Q}_{fr}(t) \quad (2)$$

$$\rho_r \cdot V_r \cdot c_r \cdot \dot{T}_r(t) = \overset{*}{Q}_r(t) + \overset{*}{Q}_{fr}(t) - \overset{*}{Q}_{out}(t) \quad (3)$$

Equations (4) to (7) describe the heat flows used in equations (1) to (3).

$$\overset{*}{Q}_h(t) = \rho_w \cdot c_w \cdot \overset{*}{V}_w \cdot [T_s(t) - T_w(t)] \quad (4)$$

$$\overset{*}{Q}_{wf}(t) = \kappa_{wf} \cdot [T_w(t) - T_f(t)] \quad (5)$$

$$\overset{*}{Q}_{fr}(t) = k_f \cdot A_f \cdot [T_f(t) - T_r(t)] \quad (6)$$

$$\overset{*}{Q}_{out}(t) = \kappa_{loss} \cdot [T_r(t) - T_{out}(t)] \quad (7)$$

The quantities $\overset{*}{Q}_r(t)$ and $\overset{*}{Q}_f(t)$ in the equations (2) and (3) represent the heat flows gained in the room and in the floor due to solar radiation, cooking, lights, etc. The parameters of this model were successfully identified using measured data. A detail description of this model and its validation can be found in [4] and [5].

Heat pump model:

Neglecting the thermal dynamics of the heat pump, both the heat flow $\overset{*}{Q}_{hp}(t)$ delivered and the electric power $P_{hp}(t)$ consumed by the heat pump are only functions of the outdoor temperature $T_{out}(t)$ and of the supply flow temperature $T_s(t)$ (see Figure 3). For the optimization of the heat flow required $\overset{*}{Q}_{req}(t)$, this information is sufficient, because for this purpose only the coefficient of performance COP as a function of $T_{out}(t)$ evaluated at a nominal supply flow temperature $\overline{T}_s$ is necessary (see Section 3.3):

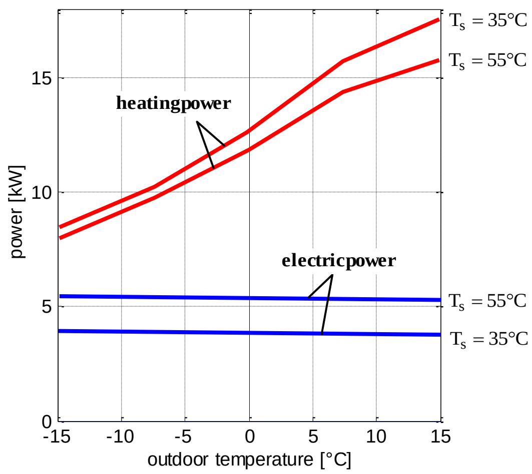

Figure 3. Power characteristic curve of the heat pump

$$\mathrm{COP}(T_{out}(t), \overline{T}_s) = \frac{\overset{*}{Q}_{hp}(T_{out}(t), \overline{T}_s)}{P_{hp}(T_{out}(t), \overline{T}_s)}. \quad (8)$$

However, in order to optimize the heating pulse duration of the heat pump with regard to its most energy-efficient operation, the thermal and electrical dynamics of the heat pump may not be neglected.

The dynamics of the heat pump for delivering the heat flow $\overset{*}{Q}_{hp}(t)$ are approximated by a first-order low-pass system with different time constants for the switching on and switching off the compressor of the heat pump. The time constant for the latter is assumed to be much smaller than the time constant for the "on" transient. The dynamics of the electrical power consumption of the heat pump $P_{hp}(t)$ are sufficiently fast by switching the compressor on and off. However, since the electrical power consumed by the heat pump also depends on the supply flow temperature $T_s(t)$, the electrical power consumption also shows a slow transient based on the transient of the supply flow temperature. This fact clearly impacts together with the transients of $\overset{*}{Q}_{hp}(t)$ the optimal duration that the heat pump should be running and thus the corresponding optimal on/off period of the heat pump. The details are explained in Section 3.3 below.

3.2 Weather Forecast

In order to predict the performance of the heat pump and the heating energy required by the house, the knowledge of the transients of the outside temperature $T_{out}(t)$ is essential. Various methods are available for obtaining the weather forecast. An external forecast from a meteorological station (e.g. MeteoSwiss for Switzerland) provides the best accuracy, but the download via internet requires a connection to the web. Since this connection is not provided in the commercial controller used in the field testing, an internal forecast is used.

The internal forecast used here is based on the simplified assumption that the shape of the temperature trajectory for the next 24 hours will remain the same as it was for the previous 24 hours. A parallel shift of the whole trajectory takes care of adjusting the predicted outdoor temperature $\hat{T}_{out}(t_0)$ to the measured outdoor temperature $T_{out}(t_0)$ at the actual time t_0. This forecast is characterized by equation (9).

$$\hat{T}_{out}(t_k) = T_{out}(t_k - 24h) + T_{out}(t_0) - T_{out}(t_0 - 24h) \tag{9}$$

3.3 Controller Algorithm

The production-type controller utilized on a standard heat pump requires the algorithm to consider the following restrictions: no sensor is available for measuring the room temperature, only the outdoor temperature, the supply, and the return flow temperatures are measured. Due to the limited computing power and memory size available, a fast algorithm was necessary.

Since an analytical solution exists for the linear quadratic (LQ) optimization that is very well known in the control theory [3], it is used here within the MPC approach to optimize the transient of heating power required over the prediction horizon. In the controller a discrete-time solution of the LQ optimization is implemented. which is summarized below. The sampling time t_{peri} used for the algorithm is found to be optimal with respect to the most energy-efficient operation of the heat pump (see section 3.4).

LQ-Optimization:
The goal is to minimize the quadratic objective function introduced in (10) for the prediction horizon $t_{hori} = N \cdot t_{peri}, N \in \mathbf{N}$ subject to the linear time-invariant model of the house introduced in section 3.1.

$$J(\overset{*}{Q}_{req}) = \frac{1}{2}\sum_{k=0}^{N}\left\{\overset{*}{Q}_{req,k}^{T} R_k \overset{*}{Q}_{req,k} + \left(T_{r,k} - T_{ref,k}\right)^T Q_k \left(T_{r,k} - T_{ref,k}\right)\right\} \tag{10}$$

The optimal solution for the controller output consists of two terms: A feed-forward and a feedback term, respectively:

$$\overset{*}{Q}_{req,k} = -R_k^{-1}B_u^T(A^T)^{-1}\cdot\left(C^TQ_kT_{ref,k} - w_k\right) - R_k^{-1}B_u^T(A^T)^{-1}\cdot\left(P_k - C^TQ_kC\right)\cdot x_k, \tag{11}$$

where w_k in the feed-forward term is recursively calculated backward over the prediction horizon t_{hori} with (12) starting from its boundary value (13)

$$\begin{aligned} w_k = &\left[A^T - A^TP_{k+1}\cdot\left(I - B_u\left[R_k + B_u^TP_{k+1}B_u\right]^{-1}B_u^TP_{k+1}\right)\cdot B_uR_k^{-1}B_u^T\right]\cdot w_{k+1} \\ &+ C^TQ_kT_{ref,k} - A^TP_{k+1}\cdot\left(I - B_u\left[R_k + B_u^TP_{k+1}B_u\right]^{-1}B_u^TP_{k+1}\right)\cdot B_v\hat{T}_{out,k} \end{aligned} \tag{12}$$

$$w_N = C^TQ_NT_{ref,N}, \tag{13}$$

and the matrix P_k appearing in the feedback term (11) as well as in the solution of w_k (12) is also recursively calculated backward over the prediction horizon t_{hori} with the discrete-time matrix Riccati equation (14) starting from its boundary value (15).

$$P_k = C^TQ_kC + A^TP_{k+1}A - A^TP_{k+1}B_u\left[R_k + B_u^TP_{k+1}B_u\right]^{-1}B_u^TP_{k+1}A \tag{14}$$

$$P_N = C^TQ_NC. \tag{15}$$

In addition, x_k appearing in the feedback term represents the vector of the state variables of the house model that are estimated utilizing a Kalman-Filter [2]. The signals used for the observer are $T_w(t)$, $T_{out}(t)$, and $\overset{*}{Q}_h(t)$. Finally, the matrices A, B_u, B_v, and C are the system matrices of the discrete-time state space description of the house model presented in section 3.1.

Weighting factors of the objective function:
Since only the ratio of the weighting factors (Q_k/R_k) in the objective function (10) is relevant for the optimization, the weighting factor for the deviation of the room temperature from the reference room temperature Q_k is set to 1. For the weighting factor responsible for the energy consumption R_k a base value of 10^{-4} is chosen. This choice smoothes the trajectory of the controller output signal $\overset{*}{Q}_{req,k}$ without any significant error in T_r.

To minimize the electrical energy consumed, the performance of the heat pump has to be considered. This is done by computing the nominal COP (8) for the prediction horizon in function of the predicted outdoor air temperature $\hat{T}_{out}(t)$ and dividing R_k at every sampling time t_k by the corresponding COP. Since the COP is insensitive to the supply flow temperature $T_s(t)$, an average value for $\overline{T}_s$ is used to determine the COP as a function of $T_{out}(t)$ only.

The tariff structure of the electric power is also considered by modifying the weighting factor R_k. For this purpose, at each sampling time t_k, the factor R_k is multiplied by the corresponding normalized tariff. For online optimization, the power cut-off times should be treated as time-variant constraints. In the analytical solution of the optimization problem introduced above, the constraints cannot be treated directly. However, setting the weighting factor R_k to a very high value for cut-off periods has a similar effect.

3.4 Pulse-width Modulation

After having optimized the heating power required over the prediction horizon, the corresponding on/off signal for the heat pump is generated by utilizing an adequate pulse-width modulation. From the point of view of energy efficiency, the optimal length of time for the heat pump to be switched on t_{on} is given by maximizing the energy ratio

$$ER(t) = \frac{\int_t \overset{*}{\dot{Q}}_{hp}\left(T_{out}(t), T_s(t)\right) \cdot dt}{\int_t P_{hp}\left(T_{out}(t), T_s(t)\right) \cdot dt}, \qquad (16)$$

rather than the coefficient of performance

$$COP(t) = \frac{\overset{*}{\dot{Q}}_{hp}\left(T_{out}(t), T_s(t)\right)}{P_{hp}\left(T_{out}(t), T_s(t)\right)}. \qquad (17)$$

The transients of both coefficients are shown in Figure 4 for a constant outside air temperature. After switching on the heat pump, the coefficient of performance $COP(t)$ increases first, reaches its maximum and decreases again, because the supply flow temperature $T_s(t)$ slowly increases.

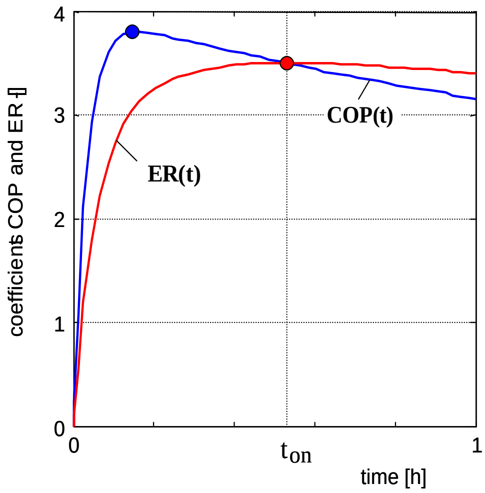

Figure 4. The transients of the coefficient of performance $COP(t)$ and the energy ratio $ER(t)$ for a constant outdoor temperature.

This in turn causes the electrical power consumption to increase (see Figure. 4) with the consequence that the energy ratio $ER(t)$ reaches its maximum while the $COP(t)$ decreases further. This maximum determines the energy-efficient length of time for the heat pump to be switched on, t_{on}.

This optimal duration is a function of the outdoor air temperature T_{out} and of the thermal dynamics of both the house and the heat pump. Figure 5 shows the optimal duration for the heat pump to be switched on, t_{on}, and the corresponding optimal on/off period, t_{peri}, as a function of the outdoor air temperature T_{out}.

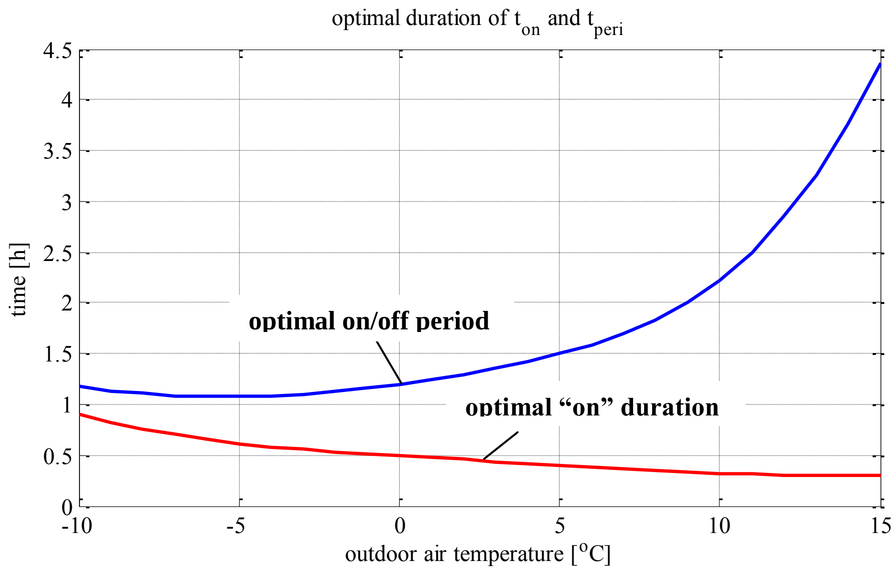

Figure 5. Optimal duration for the heat pump to be switched on t_{on} and the corresponding optimal on/off period t_{peri} as a function of the outdoor air temperature

The relation between t_{peri} and t_{on} is given by (18), which defines the heating energy delivered during t_{on} to be the same as the optimized heating energy required for one period duration t_{peri}.

$$\overset{*}{\dot{Q}}_{req} \cdot t_{peri} \overset{!}{=} \int_{t_{on}} \overset{*}{\dot{Q}}_{hp}\left(T_{out}(t), T_s(t)\right) \cdot dt \, . \tag{18}$$

Figure 4 clearly shows that the transient for the energy ratio $ER(t)$ is flat around the optimal t_{on}. This fact permits a wide range of choices for t_{on} without any significant loss in efficiency. Due to this insensitivity, t_{peri} may also be chosen to be constant, which is optimized for a nominal operating point.

Using the on/off signal defined in (19) ensures that the required amount of heating energy is effectively delivered:

$$u_{hp} = \begin{cases} 1 & \text{if} \int_{t} \overset{*}{\dot{Q}}_{hp}(t) \cdot dt \leq \overset{*}{\dot{Q}}_{req,0} \cdot t_{peri} \\ 0 & \text{else} \end{cases} \tag{19}$$

3.5 Simulation Results

A one-year simulation has shown that the desired comfort level can be achieved very well, combined with an economical operation of the heat pump. For this simulation, the average room temperature was $\overline{T}_r = 20.2\,°C$, with a standard deviation of $\sigma(T_r) = 0.2\,°C$. The set-point $T_{ref}(t)$ for the room temperature was required to be at $20.0\,°C$. Only days with an average outside temperature of equal to or less than $15\,°C$ were taken into account. With the MPC controller, 62% of the heating energy required was produced in low-tariff periods, i.e., in 12.4 hours per day (average).

By comparison, a heat pump equipped with a standard relay-type controller delivered only 40% of the heating energy in the low-tariff periods, without any advantage in comfort ($\overline{T}_r = 20.0\,°C, \sigma(T_r) = 0.3\,°C$). Using the MPC controller thus decreased the operating costs by 10%.

Figure 6 shows the simulation results for one week in March of 1999, using the MPC controller. Measured outdoor temperature and solar radiation (measured in Kloten, Switzerland, 1999 by MeteoSwiss) are used for this simulation.

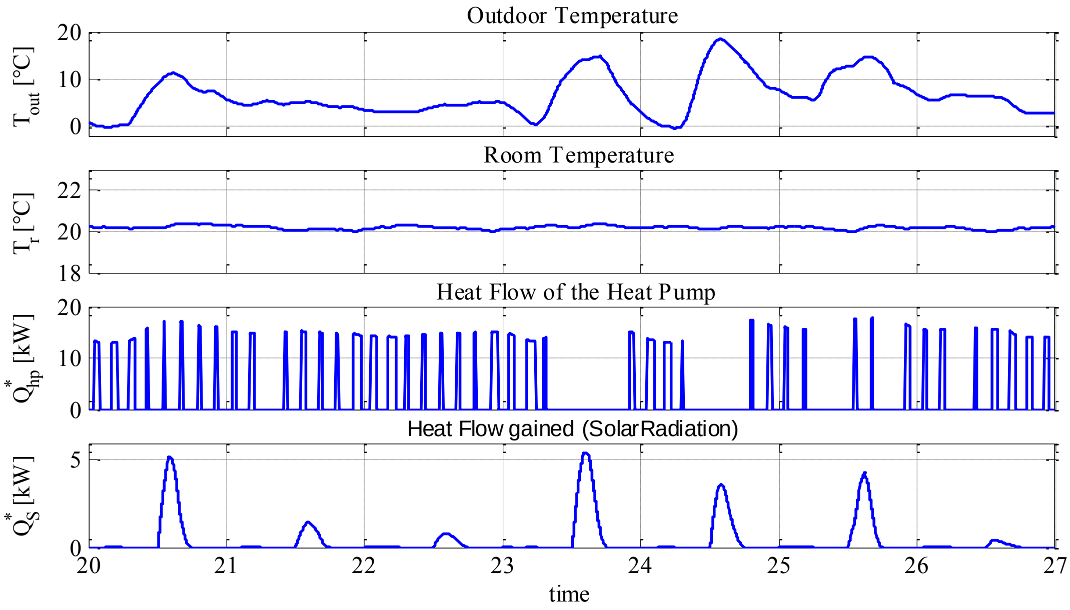

Figure 6. Simulation results for one week in March 1999.

3.6 Field Testing Results

The MPC algorithm introduced in Sections 3.3 and the pulse-width modulation introduced in section 3.4 were implemented in a production-type controller of a heat pump and tested for three weeks in an occupied single family house. The measurements confirm the simulation results. An average room temperature of $\overline{T}_r = 20.2\,°C$ was achieved (set-point $T_{ref}(t) = 20.0\,°C$), with a standard deviation of $\sigma(T_r) = 0.4\,°C$. Measured data for one week are shown in Figure 7.

During the test period it was noticed that for some unknown reason the heat pump delivered only 80% of its nominal heating power. Although the model of the heat pump was not modified to take this error into account, due to the observer the room temperature was still held at its set-point. One consequence of the uncertainty in the heat pump power characteristics was that too much heating energy was produced in high-tariff periods. The optimization result requires as much heating energy as possible to be supplied during the low-tariff period just prior to the beginning of the high-tariff period. Since the heat pump was unable to produce all the required heating energy in that period, the rest had to be produced during the subsequent high-tariff period.

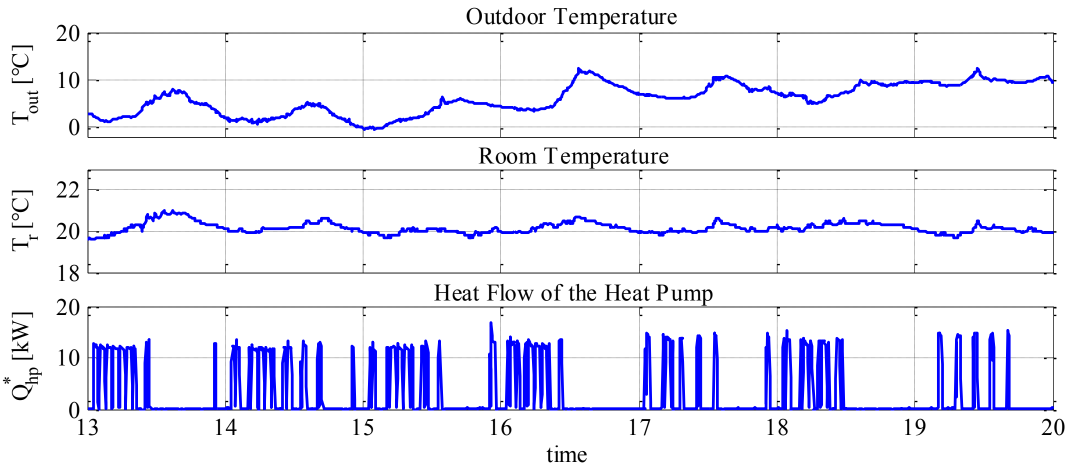

Figure 7. Simulation results for one week in March 1999.

4. CONCLUSIONS

Two concepts of a new control approach developed within the scope of the research project called "Pulse-width modulation for heat pump heating systems" funded by the Swiss Federal Office of Energy are introduced. In contrast to the conventional controllers in both concepts, the heating energy required by the house is directly quantified and delivered into the house in appropriate portions.

This approach provides a more effective way of taking into account the power tariff structure (low and high tariff periods) and the power cut-off times than conventional approaches can. Especially in the more sophisticated concept based on MPC (Model Predictive Control), the weather forecast as well as the thermal dynamics of the house may also be considered in order to optimize the size and the distribution of the heating energy portions with respect to the most efficient operation of the heat pump.

The simulation results for the MPC control concept show a very good tracking of the desired room temperature and a significant reduction of the electrical power costs without any loss of comfort.

Although the algorithm for the linear quadratic (LQ) optimization utilized in the MPC control concept (see section 3.3) seems to be complex, it could easily be implemented in a production-type controller and tested successfully as a prototype controller in a single-family house.

ACKNOWLEDGMENTS

The results reported in this paper have been obtained within the scope of the research project called "Pulse-width modulation for heat pump heating systems" funded by the Swiss Federal Office of Energy (BfE) [6]. The financial support of the BfE, through the research program UAW, is gratefully acknowledged.

REFERENCES

[1] E. F. Camacho and C. Bordons. *Model Predictive Control*. Springer, London, 1999.

[2] H. P. Geering. *Regelungstechnik*. Springer-Verlag, Berlin Heidelberg, fünfte Auflage, 2000.

[3] B. D. O. Anderson and J. B. Moore. *Optimal Control: Linear Quadratic Methods*. Englewood Cliffs: Printice –Hall 1990.

[4] E. Shafai, S. Ginsburg, R. Wimmer, and D. Zogg. *Validation of a model for a residential heat pump system based on short-period measurements*. Proceedings of the Sixth International Energy Agency Heat Pump Conference: Heat Pumps – a Benefit for the Environment, volume Poster Presentations., Berlin, June 1999.

[5] G. Reiner, E. Shafai, R. Wimmer, D. Zogg, H.R. Gabathuler, H. Mayer, and H.U. Bruderer. *Kurztestmethode für Wärmepumpenanlagen. Phase 1 bis 3: Messung, Modellierung und Erprobung der Parameteridentifikation*. Schlussbericht ENET-Nr. 965740, Bundesamt für Energie, Bern, November 1998.

[6] H.R. Gabathuler, H. Mayer, E. Shafai, and R. Wimmer. *Pulsbreitenmodulation für Klein-wärmepumpen. Phase 1*. Zwischenbericht ENET-Nr. 9763664, Bundesamt für Energie, Bern, Juni 1998.