Dynamic Determination of Ground Thermal Conductivity and Minimum Thermal Response Test Duration Using the Line Source Model

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Abstract

Ground thermal properties, such as the undisturbed ground temperature and the effective thermal conductivity along the depth of a vertical borehole, play an important role in designing a proper ground heat exchanger (GHX) for ground source heat pump (GSHP) applications. The effective ground thermal conductivity (GTC) is usually determined with a combined approach of thermal response test (TRT) an inverse modeling. Among existing models for analyzing the TRT data, the line source model is most widely used in practice because it is easy, fast and straight-forward to implement. However, the current practice requires extremely stable power supply for 48 hours during a TRT. In order to reduce the test duration, and the associated cost, while retaining a reasonable accuracy, an algorithm is developed to dynamically analyze the TRT data during the test and determine the minimum test duration. It is found that, for TRTs with stable power supply, the test duration could be shortened by 20 to 40% with this algorithm while retaining the GTC value within ±5% uncertainty of that determined with 48 hours or longer TRT data.

Keywords: ground thermal conductivity; thermal response; line source model; ground source heat pump

1. Introduction

The design of a Borehole Heat Exchanger (BHE) based Ground Source Heat Pump (GSHP) system is a systematic and challenging work. The success of its design depends heavily on the knowledge of many aspects, including the operating characteristics of the heat pump, building heating and cooling demands, as well as the thermal properties of the ground, e.g. thermal conductivity, thermal diffusivity, and undisturbed ground temperature. When it comes to the vertical BHE, the situation becomes more challenging as the thermal properties of the ground vary with respect to depth.

Thermal response test (TRT) combined with analytical or numerical modeling is often used to determine the ground thermal conductivity (GTC). Among the existing analytical or numerical models, the line source model is most widely used in practice because it is easy, fast and straight-forward to implement. More importantly, it is

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capable of providing reasonably accurate result provided all the requirements for the TRT test are strictly followed, such as 48-hour test duration with extremely stable power supply.

The work presented in this paper is to (1) review the existing analytical or numerical models with regard to requirements for the TRT, (2) scrutinize the necessity of the stringent requirements for the TRT associated with the line source model analysis, and (3) explore possible ways to reduce the TRT duration, thus cost, while retaining acceptable accuracy for the measurement of the GTC.

2. Literature review

2.1. Experimental test

TRT has become a widely used approach to obtain soil thermal properties [1]. The concept of TRT is quite simple and can be described as follows: a pilot borehole is first drilled at the field with approximately the same diameter, depth, and interior design of the ground heat exchangers (i.e., pipe size, material, and layout, as well as grouting material) planned for the site. It is followed by injecting heat to or extracting heat from the ground, and necessary data, e.g. heat flux, temperature of fluid entering and leaving the GHX, are collected during the test, which usually takes 48 hours. The TRT captures the combined effects of various parameters that affect the heat transfer in the ground formation, such as the thermal conductivities of different soils and rocks along the depth of the borehole, and the underground water movement. Therefore, the TRT result is very useful to determine the effective GTC along the depth of the borehole [1-4].

In addition to characterizing the thermal response of the ground formation, the TRT data can also reveal the undisturbed ground temperature. Besides, the geological information obtained through drilling the pilot borehole is valuable for selecting appropriate drilling technique and equipment for the particular site [5].

Like any other experiments, a certain number of requirements or procedural instructions need to be followed to obtain valid results. Fail to do so might lead to misleading values of the GTC and the consequence could be an underperforming or failed GSHP system, or an over-sized and expensive GHX. The most dominant requirements include test duration, power supply stability, and cutting time for data analysis. Those requirements are also of great practical interests as they relate directly to the cost of the experiment.

Generally, longer test time contributes to obtaining more accurate thermal properties of ground with higher confidence. However, it is also associated with higher cost and higher possibility of encountering external disturbance (e.g., heat loss above the ground due to loss of insulation). Consequently, obtaining accurate GTC with minimum test time exhibits a practical interest. Nevertheless, the required test duration is usually specified as a range and remains a debating subject in literature. John Geyer suggested that the power levels and supply and return water temperatures should be sampled every 5 minutes for 40 to 48 hours [5]. The ASHRAE applications handbook [6], however, suggested that the test data should be collected at least every 10 minutes for a duration of 36 to 48 hours . Austin et al. also suggested a test length of 50 hours [7]. It may not be possible to suggest a single test duration for all the TRT tests since the thermal response of a BHE is affected by many factors, including the heat transfer within the borehole and the thermal conductivity of the ground formation, and on the other hand, the test duration is also determined by the model used to interpret the measured TRT data. According to literature, the line source model requires minimum test duration of 40 to 50 hours [5, 6], other models that can simulate the transient heat transfer process inside the borehole is capable of reducing the minimum test duration [8-14], and it is reported that a comprehensive 3-D numerical model can reduce the needed time duration to 12 hours in some circumstances [15]. Nevertheless, those statements are generally concluded based on limited study of a few cases and it is not clear whether the shortened test duration is applicable to other cases.

Another important requirement is the high stability of power input. it is suggested that the relative standard deviation of input power should be less than ±1.5% and the peaks are less than ±10% of average [6]. Also, in order to use a simple analytical method like the line source model to interpret the measured data, at least 10 to 18 hours of the earliest data need to be discarded [6]. Again, the recommended cutting time is also a range and the exact cutting time could be different for different TRTs.

2.2. Analysis models

Line source model has been the most widely used model in practice for interpreting the TRT data. The original line source model was developed by Lord Kelvin [16]. Ingersoll and Plass first applied the model on GHXs [17, 18]. This model approximates the vertical borehole as an infinite line source and thus neglects the heat transfer at the end of the borehole. The ground is assumed to be an infinite medium with a uniform initial temperature. The heat conduction process is simplified as a one-dimensional case by neglecting the heat transfer along the borehole axis.

According to the line source model, the thermal response of the ground to a constant heat flux is expressed as:

$$T_f(t) - T_0 = \frac{q}{4\pi k} In(t) + qR_b + \frac{q}{4\pi k}\left(In\left(\frac{4\alpha}{r^2}\right) - \gamma\right) \quad (1)$$

Where $T_f(t)$ is the measured average temperature of the fluid in the GHX at time t, T_0 is the undisturbed ground temperature, r is the borehole radius, γ is the Euler's constant, q is the heating/cooling rate per unit length of the line source, R_b is the borehole thermal resistance, k and α are thermal conductivity and diffusivity of the ground. The above equation is valid only when the time scale is greater than $5r_b^2/\alpha$ [19].

As can be seen from the above equation, the average fluid temperature resulting from a constant heat flux is linearly related to the natural logarithm scale of time. GTC can thus be determined from the slope of the linear relationship.

Currently, the line source model is the most widely used model to interpret the TRT results and estimate GTC [17, 18, 20]. This model has some limitations due to the simplification of the physics. First, this model is incapable of modeling the heat transfer within the borehole and thus the measured data during the first 10-18 hours of the test need to be discarded. Second, any significant fluctuation in heat injection/extraction rate invalidates the model as demonstrated by Austin [21] and therefore the heat flux (usually provided with an electric resistance element) needs to be extremely stable [22-24].

Many attempts have been made to improve the line source model by approximating the physics more accurately. The finite line source model [25] and cylinder source model [26] represent the vertical borehole with a line source with a certain depth and an infinite cylinder source, respectively. However, both models seem to be less appealing in practice as no significant improvements on calculation accuracy have been reported while increasing the complexity of the model and cost of computation. Moreover, since both models discard the intrinsic effects of heat transfer inside the borehole and they are all based on the assumption of constant heat flux along the borehole, they have the same stringent requirements on the power stability and test duration as the infinite line source model does.

Another notable attempt is by exploring thermal resistance and capacity model (TRCM) approach, which certainly has become a trend recently [8-14, 27]. The basic idea is to represent the physics with electrical circuits which contains multiple resistances and capacitances. By positioning those resistances and capacitances appropriately, the intrinsic heat transfer process can be captured [8-14]. Consequently, TRCM has a great potential of handling varying heat flux and shortening TRT test duration. However, there is few literature reporting the efficacy of using TRCM to determine GTC with the TRT data.

The methodology of combining 2D or 3D numerical model with a parameter estimation technique has also emerged owing to the advancement of computation capacity. It has been reported that a 12-hour TRT is enough to determine the GTC with acceptable accuracy using this approach when some assumptions are appropriate [15]. The main downside of this approach, however, is the prohibitively expensive computation cost, e.g. expensive hardware/software and long computation time, which has prevented its wide spread application. Another challenge of this approach is that the same numerical model needs to be used in the GSHP design and simulation program in order to accurately predict the GHX temperature resulting from the heat extraction/rejection loads of the building with the determined GTC value. Given the complexity of the numerical models, it is usually not practical to integrate them within the GSHP design and simulation program, which needs performing hourly simulations for multiple years.

Miaomiao He [28] conducted sensitivity studies on the influence of several factors, e.g. cutting time, test duration, fluid circulation flow rate, grout conductivity and heterogeneity of ground, on the GTC determined using the line source model. The results showed possibility of reducing test duration while obtaining GTC with acceptable accuracy for particular cases. Poulsen and Alberdi-Pagola [29] later proposed a method to determine the minimum test duration during a TRT, of which the measured data are analyzed with the parameter estimation method. However, this method is not applicable for the TRT analyzed with the line source model.

3. Impacts of cutting time and duration of TRT data on GTC evaluated with line source model

In this work, the possibility of relaxing the requirements of TRTs analyzed with the line source model is explored. A new algorithm has been developed in this work to automatically determine the needed test duration of ongoing TRTs that can yield a GTC value with the same uncertainty of that determined with a longer (i.e., 48-hour) test.

The impacts of cutting time and duration of TRT data on GTC evaluated with line source model are investigated with a 48-hour TRT data set measured in Oklahoma City (OKC), Oklahoma [30]. The GHX input/output temperature, heat flux, flow rate data was collected and stored every minute. As shown in Figure 1 (a), the DeltaT, i.e. the difference between the measured mean fluid temperature and the undisturbed ground temperature, was plotted against time with natural logarithm scale. The plot shows a clear linear relationship between the DeltaT and the time in natural logarithm scale, except a jump at about the 2rd hour.

The line source model was used to calculate GTC based on the TRT data. A sensitivity study of using different cutting point and duration of the dataset was conducted, and a matrix of resulting GTC values are obtained and plotted in figure 1 (b) (the values are indicated by different colors). By applying conventional cutting point and duration, e.g. cutting point at 10 hours and duration at 20 to 35 hours, the GTC is calculated to be 1.75 Btu/(hr-ft-F) with an estimated error of ±5% [31]. As shown in this figure, most of the obtained GTC values fall into the range between 1.7 and 1.8 Btu/(hr-ft-F). In current case, the minimum test time (cutting point + duration) needed to obtain a GTC value same as that obtained with the full data set can be as short as 15 hours (cutting point at 5th hour with a duration of 10 hours), which is much shorter than the suggested test time, i.e. 40-50 hours[5-7][5-7][5-7][5-7][5-7][5-7]. Similarly, in this particular case, the cutting point can be much shorter, e.g. 5 hours, than that has been suggested in literature i.e. 10 to 18 hours.

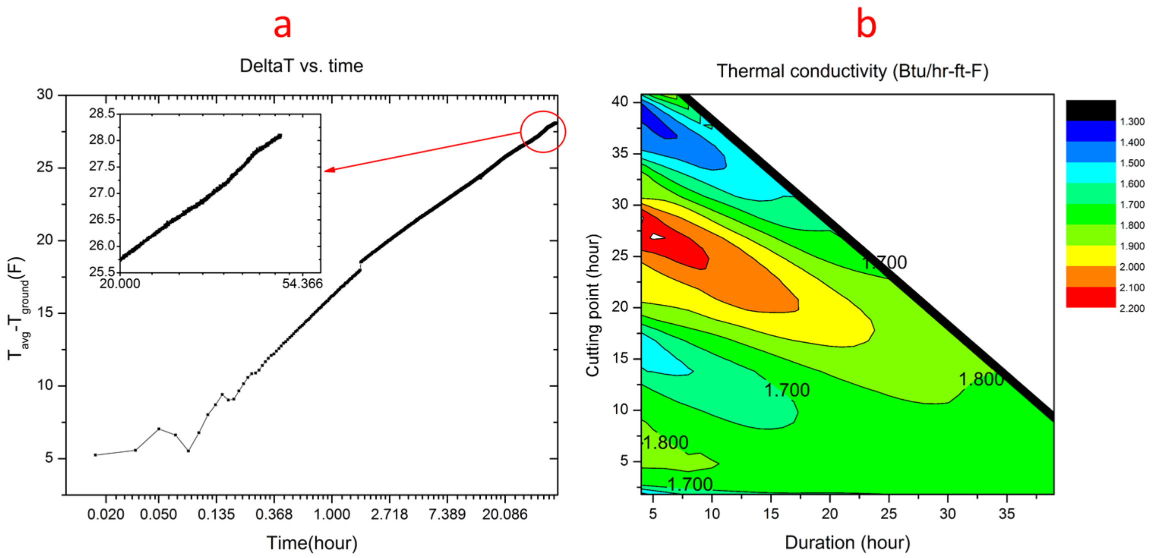

Fig. 1. (a) Difference between the measured average GHX temperature and the undisturbed ground temperature with respect to time on a natural logarithm scale; (b) The obtained ground thermal conductivity values with different combination of cutting point and duration

Strong variation is observed for GTC values obtained while using short duration and late cutting point—a minimum value around 1.3 Btu/(hr-ft-F) is obtained when using a cutting point at the 27th hour and a duration of 5 hours, while a maximum value around 2.3 Btu/(hr-ft-F) is obtained when using a cutting point at the 37th hour and a duration of 5 hours. The possible cause might be due to the fluctuation of DeltaT at late part of the test, i.e. 25th to 40th hour, as shown in Figure 1(a). In situations when the cutting point is beyond the 25th hour, longer test becomes necessary to compensate the regional fluctuation in the test data to obtain a better GTC value, e.g. 48 hours as recommended in other references[5-7][5-7][5-7][5-7][5-7][5-7].

The quality of linear fitting at different cutting points and durations are further examined by calculating the R-squared (a statistical measure of how close the data are to the fitted regression line), which shows in Figure 2. It is further verified that the quality of the linear fitting is high, i.e. the overall R-squared are bigger than 0.955. The quality of linear fitting degrades as the cutting point is later than the 25th hour and the duration is short, e.g. 5 hours.

Apart from the minimum test time requirement, stable power supply/heat flux is another strict requirement for the TRT. The ratio between the standard deviation and the average of the heat flux are calculated to examine the stability of the power supply, as shown in Figure 3. The power supply is proved to be highly stable as the ratio is significantly smaller than the requirement, i.e. 1.5% [6]. The highest value of the ratio, i.e. 0.3%, appears when the cutting point is around 7th hour while the duration is about 17 hours. Consequently, it is confident to conclude that the negative effect of the variation in power supply is minimal to the overall test for this particular case. The fluctuation of the DeltaT at the late time of the test must be due to external causes, e.g. heat loss at the pipes above the ground due to loss of insulation.

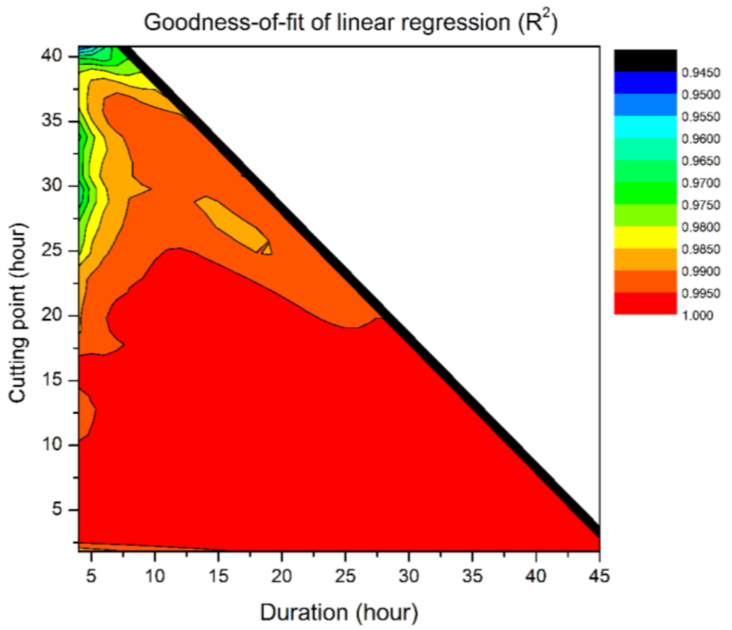

Fig. 2. Goodness of linear fitting (R2) with respect to cutting point and duration

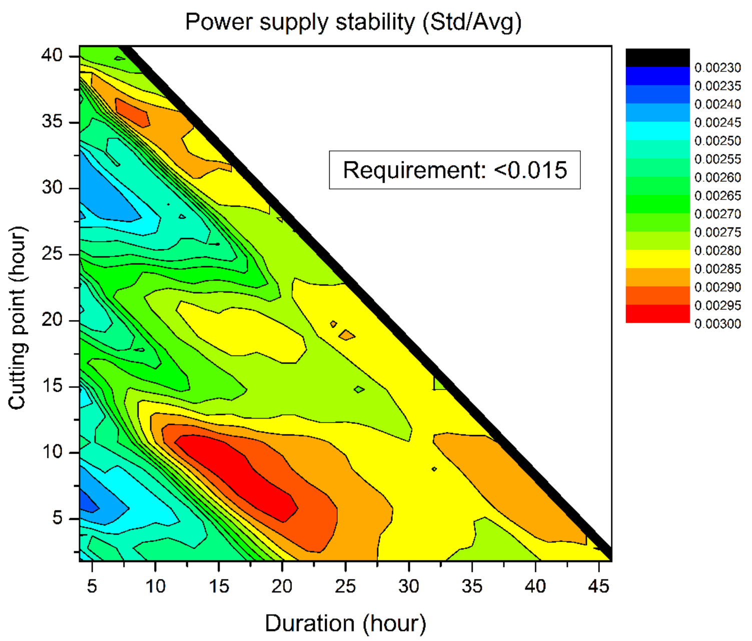

Fig. 3. Power supply stability (std/avg) with different cutting points and durations

4. A new algorithm to determine minimum test time of TRT

The result in the above case study suggests the possibility of shortening the test time of TRT. Considering that the longer test time often associates with higher cost and increased chance of encountering external disturbance, it is of great practical benefits to avoid the unnecessarily long test time while obtaining a GTC value with acceptable accuracy.

A new algorithm is developed to determine the minimum test time during an ongoing TRT. To implement this algorithm, a TRT needs to be proactively monitored and the GTC will be evaluated with the line source model at the real-time with the measured data during the test. The TRT can be aborted immediately when the GTC value obtained from the measured data is converged within the uncertainty of typical TRTs, e.g. 5% [6]. The flow chart of the new algorithm is shown at Figure 4 (a). The step-by-step procedure of the algorithm is explained below:

- Set the cutting point at the 10th hour as recommended by the ASHRAE applications handbook [6].
- After the cutting point, evaluate GTC with the following 1, 2, … and 10 hours of TRT data.
- Calculate two parameters based on the above evaluated 10 GTC values—the relative standard deviation (ε_1) and the relative variation (ε_2)—and compare them with a set of criteria. If the criteria are satisfied, the TRT can be stopped and the GTC value is reported; otherwise, the process is iterated until the criteria are satisfied, or the total test time is more than 48 hours. ε_1 and ε_2 are calculated as expressed in Equations (1) and (2), respectively:

$$\varepsilon_1 = std(k_{i+j})/mean(k_{i+j}) \tag{1}$$
$$\varepsilon_2 = (\max(k_{i+J}) - \min(k_{i+j}))/mean(k_{i+j}) \tag{2}$$

where, k_{i+j} is the GTC value evaluated with TRT data within various durations, i is the iteration index, and $j = 1, 2, \dots, 10$.

This algorithm was applied to the OKC TRT data to check its validity. The result is shown in Figure 4 (b), where the green point represents the minimum duration after the cutting point and the resulting GTC value determined by the algorithm. For this specific case, the test time can be shortened by 5 hours with the new algorithm while the calculated GTC value is only 2.2% higher than that determined with all the 38-hour TRT data after the cutting point at the 10th hour. The saved time, however, is not as significant as observed previously. It is due to the fact that the thermal conductivity of the grout used in this case, i.e.0.43-0.45 Btu/(hr-ft-F), is significantly lower than that of the ground, i.e. 1.75 Btu/(hr-ft-F), the profile of the GTC values determined with various durations exhibits a long transition period until it finally converges, as shown in Figure 4(b).

This algorithm is further applied on and validated against another 9 TRT datasets. A summary of those TRT cases is shown in table 1 and the validation results are plotted in Figure 5.

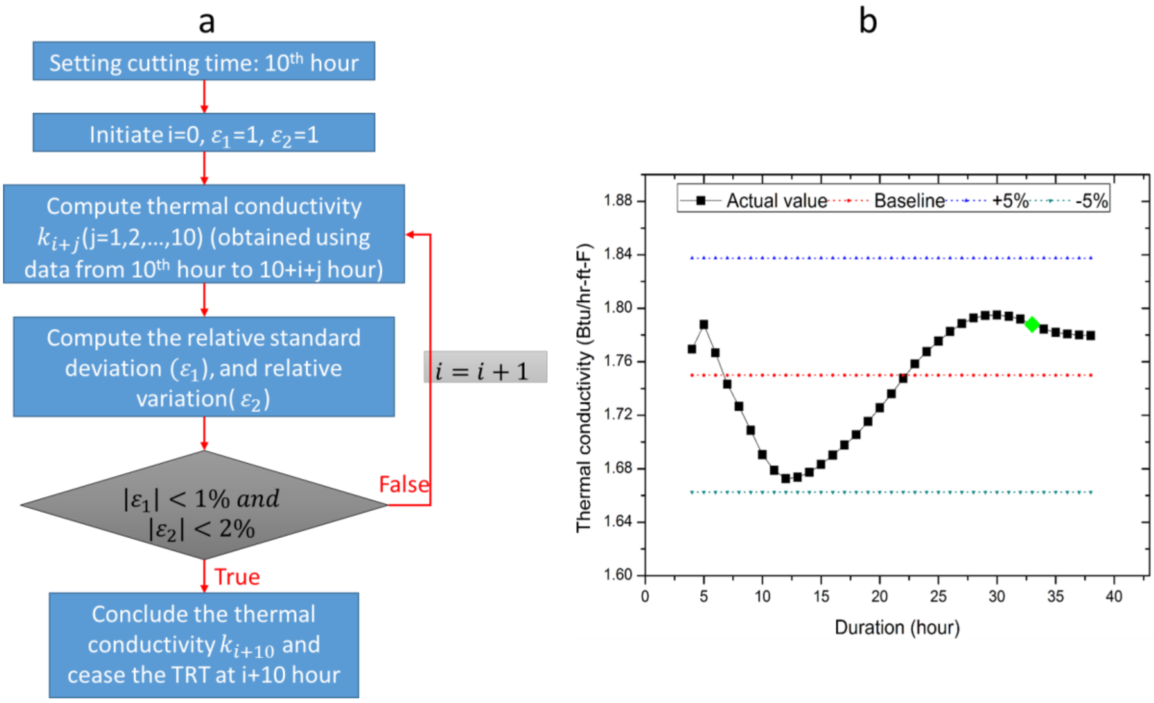

Fig. 4. (a) A flow chart of the new algorithm for determining the minimum test time, and (b) its application on the OKC TRT data

Table 1. A summary of the 9 TRT cases [32]

Case	Location	Borehole diameter (inch)	Grout thermal conductivity (Btu/hr-ft)	Depth (ft)	Duration (hour)	Heat rate (Btu/hr-ft)	Measured thermal conductivity (Btu/hr-ft-F)	Undisturbed ground temperature (F)
1	Brighton, CO	5.5	1.07-1.2	605	43.6	87.4	1.16	53.5-60.2
2	Grand Junction, CO	5.5	1.4	605	47.7	56.5	1.43	59.3-66.4
3	Carlin, NV	5.25	0.43	265	41.4	65.8	1.32	57.1-61.7
4	Carson City, NV	5.25	0.43	300	46	68.3	0.83	63.9-107.7
5	Dugway proving ground, UT	5.25	0.43	220	40.9	65.7	0.59	55.3
6	Ogden, UT	5.25	0.93	350	43.5	63	1.71	55.7-57.4
7	Salt Lake City, UT	5.25	1.1	420	44	69.3	1.02	na
8	Bluffdale, UT	5.25	1.07	410	44	70.7	1.04	55-56.8
9	Jackson, WY	5.25	0.4	355	41.6	62.6	1.31	50.1-51.5

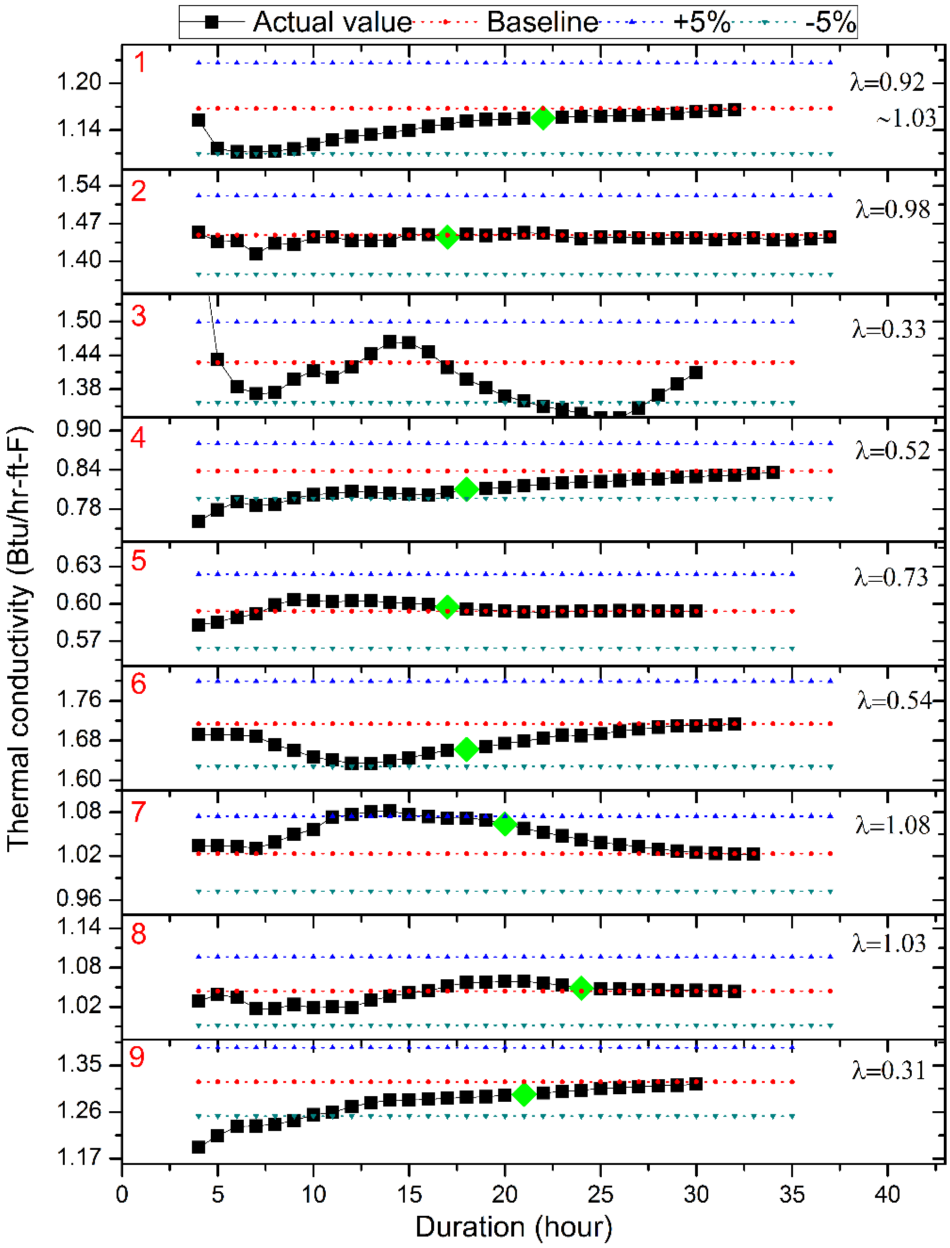

Fig. 5. The application of the developed algorithm on the 9 TRT cases

In Figure 5, the black dots represent the GTC values evaluated using the line source model, i.e. cutting point at 10th hour and using TRT data within a certain time period after 10th hour (i.e. 5 hours, 10 hours, etc.), while the green dots represent the minimum duration and the resulting GTC value determined by the new algorithm. It is clearly shown that the GTC values determined with the new algorithm is very close to those determined with the full data set except for case 3. It turns out that the power supply for case 3 is very instable, which invalidates the line source model. As shown in Figure 5, a general 20% to 40% cut in the test time is achieved while retaining the GTC value within ±5% uncertainty of that determined with 48 hours or longer TRT data.

This algorithm can also act as an indicator for poor heat flux quality, or other factors that invalidate the line source model. As shown in the Figure 5, the algorithm was unable to converge on case 3 when the heat flux is instable. As also clearly shown in this figure, the trend of the GTC values evaluated with various durations is significantly affected by the ratio between grout and ground thermal conductivity λ (k_{grout}/k_{ground})—when the ratio is bigger than 1, the trend depicts an ascending profile as shown in cases 4, 6, 9, and vice versa; while the ratio is close to 1, the trend, on the other hand, depicts a flat line as shown in cases 2 and 5.

5. Conclusions

In this work, the line source model and its requirements for the associated TRT are scrutinized, and more importantly, critiques are given to the necessity of those strict requirements. It is found that the needed minimum test time varies depending on many factors, and it would be difficult to suggest a single value for all TRTs. The existing stringent requirement on the test time could lead to unnecessary long test hours, and thus the cost, in some cases.

A new algorithm that enables a TRT tester to determine the minimum needed test time during the process of a TRT is developed. This algorithm has been tested with 10 TRT datasets. The results indicate that the algorithm is capable of reducing the overall TRT test time by 20 to 40% in general while obtaining a GTC value within ±5% uncertainty of that determined with 48 hours or longer test data. Also, the results exhibit the possibility of using the developed algorithm as a diagnosis tool for the quality of the TRT data. It has also been found that the ratio of grout and ground thermal conductivity λ (k_{grout}/k_{ground}) has a significant impact on the minimum test time of a TRT.

6. Copyright

Acknowledgements

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