Nonequilibrium unsteady thermodynamic and spatio-temporal properties of ground source heat pumps

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Abstract

A comprehensive energy analysis was performed on a strongly nonequilibrium unsteady thermodynamic system comprising ground/soil, borehole heat exchanger (BHE) and ground source heat pump (GSHP) in vertical geothermal systems (VGS). The analysis was based on development of an unsteady theory for VGS, taking into account both spatial and temporal variability of ground temperature and the energy characteristics of the heat pump.

The analysis showed that all energy characteristics of VGS can be determined by two similarity parameters: α_{sb}, the thermal diffusivity ratio of soil and brine, and k_{sb}, the thermal conductivity ratio of soil and brine. The mathematical model developed was used to study quantitative and qualitative characteristics of extractable energy. In particular, the correlation between the amount and quality of the energy produced by VGS was analysed in detail.

The fundamental energy characteristic of an energy well for a given VGS was developed. It was shown that this characteristic completely defines the quantitative and qualitative energy characteristics in time for any given VGS and depends on three parameters.

These results allowed geometric and energy parameters of the energy well to be linked to the coefficient of performance (COP) of GSHP. It was demonstrated that the energy efficiency of each individual GSHP system requires its own analysis and that the suggested approach can be used for optimisation of GSHP systems. This enables innovative strategies for future VGS design.

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Keywords: vertical geothermal system; strongly non-equilibrium thermodynamic systems; optimal GSHP systems; thermodynamical scheme of VGS; energy discharge of BHE; similarity and dimensional methods;

1. Introduction

Implementation of heat pumps (HP) has long been considered an important part of the sustainability transformation of energy systems. The significance of heat pumps for transition towards a 'zero emissions society' is now being

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discussed in light of the development of the smart grid, which allows integration of renewable energy into the system and use of heat pumps that consume 'green' electricity for heating and cooling [1]. Moreover, residential heat pumps are themselves seen as important elements in the smart grid because of their capacity to provide system balance services [2]. However, to capitalise on heat pumps as 'flexible agents' in the energy system, it is necessary to ensure a sufficient number of installations [1, 2].

Among the different types of HP available, ground source heat pumps (GSHP) deliver the largest energy savings [3] and are seen as one of the most efficient, comfortable and quiet heating and cooling technologies available today [4,5]. A distinctive feature of GSHP systems is their reliability, *i.e*. they generate the required amount of thermal energy (heat/cold) irrespective of ambient conditions. Besides, GSHP have low operating costs [6-8]. However, HPs of this type generally require higher initial investment than other HP types, which can be explained by the high cost of ground source exchangers (GSE) [9]. Despite this, the GSHP market is growing rapidly, with the number installed having increased by more than 8% annually since 2010, and the annual growth rate in GSHP energy use is over 10% [10]. The geographical coverage of GSHP is also expanding, from 26 countries in 2000 to 48 countries in 2015 [11].

In the present analysis we considered vertical GSE or borehole heat exchangers (BHE) which, unlike horizontal GSE, do not require large installation areas. However, BHE are expensive and therefore accurate dimensioning is critical for reducing the investment costs, thereby improving the economic attractiveness and competitiveness of GSHP with BHE (*i.e*. vertical geothermal systems, VGS) compared with other types of HPs [12].

Currently, two branches in theoretical and experimental studies of VGS can be identified:

- A branch focusing on analysing heat transfer in borehole heat exchangers (see [12] and [13]). Essentially, this concerns research on special heat exchangers, ensuring the extraction of geothermal heat from the Earth by secondary fluid as the energy carrier. Such heat exchangers have different design features, the most common being a U-tube BHE. An overview of various types of BHE can be found in [14]. The theoretical basis for such studies is Kelvin's theory of heat source and Laplace transformation [15,16] and their various modifications [17-19]. These models are fundamental for organisation and interpretation of the results of the thermal response test (TRT) [20], which is used for experimental determination of ground thermal conductivity k_s, ground undisturbed temperature T_0 and borehole thermal resistance R_b as basic parameters for VGS simulation *in situ* using a mobile testing facility [21].
- An alternative branch involving development of design/simulation software for VGS, based on typical simulation models and use of experimental data from TRT (see [12] and [22]). The main purpose of these models is determining the length of BHE providing the necessary amount of extractable geothermal heat for the operation of a heat pump (HP) of given power. It should be noted that in the current framework of VGS research, a source of energy (BHE) is considered independently of the features of the HP and its operation, which is a consequence of the assumption of constancy of heat flux per unit length of BHE.

In previous work [23], we developed a stationary model of VGS by considering it as a single, strongly nonequilibrium thermodynamic system consisting of a soil (energy source), a borehole with secondary fluid as an energy carrier (energy well) and a GSHP as a converter of low-temperature energy into high-temperature energy. This model uses continuity conditions for temperature and local heat fluxes between VGS components. The results of that study showed nonlinear dependence of BHE capacity on its length, which (in contrast to conventional models) allows new approaches to optimise the length of BHE to be proposed.

In real conditions, VGS operation is a non-stationary process of extracting of geothermal heat due to changes in, among other factors, weather conditions and activities of consumers, which define "external" time scales. VGS has its own time scales associated with design features of the BHE and ground/soil type. Therefore, modelling non-stationary processes of VGS is one of the possibilities for VHS optimisation in terms of getting energy of the required amount and quality with minimum BHE length.

The objective of the present study was to elaborate an unsteady phenomenological model of VGS and study its spatio-temporal properties, in order to enable new approaches for developing optimum strategies for design, control and operation of VGS.

2. Problem statement

An essential task in the design of VGS is calculating the depth of BHE necessary for generating the required amount of thermal energy. To this end, it is essential to consider the thermodynamic interaction of all elements of the ground/soil, BHE and HP, which determines the energy exchange between the ambient, energy well and HP evaporator. In the following, we consider a VGS consisting of soil (energy source), borehole (energy well) and GSHP (energy converter).

The reliability and stability of VGS is mainly determined by the capability of the energy source (ground + BHE) to generate energy in the required amount and quality over a long period (heat during the cold season and cold in summer, with a given HP inlet temperature). Therefore, modelling and dimensioning of BHE for VGS are crucial stages in the design of GSHP systems, affecting both their reliability and cost. Optimising a BHE to ensure its capacity at minimum length would make VGS more competitive with other types of HP systems. Such optimisation requires construction of phenomenological models (without empirical parameters) of energy exchange between the components of the VGS with essentially different heat transfer mechanisms and development of analytical tools for studying heat transfer in VGS. This would allow reliable design software for optimal dimensioning of BHE for a given VGS to be developed. Figure 1 presents a schematic diagram of a VGS.

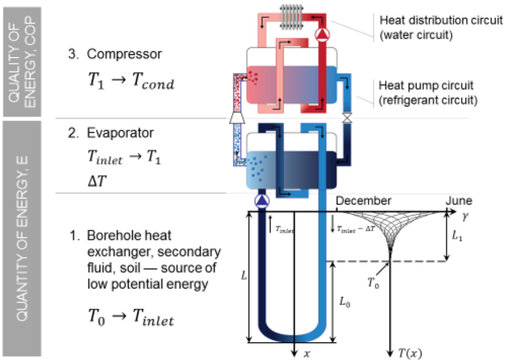

Figure 1. Schematic diagram of a vertical geothermal system (VGS).

Thermodynamically, the VGS can be divided into three components with essentially different energy transfer mechanisms:

1. The **BHE,** with secondary fluid as a heat exchanger and soil as a source of geothermal energy. In combination, they represent a source of low-temperature energy for a heat pump with molecular heat transfer in soil and molecular-convective heat transfer in the BHE. Soil is assumed to be homogeneous and isotropic. In this component of VGS, geothermal energy is extracted from soil with undisturbed temperature T_0 and transferred by the secondary fluid. The inlet and outlet temperatures of the HP evaporator are denoted T_{inl} and $T_1 = T_{inl} - \Delta T$, respectively, where ΔT is the temperature difference across the evaporator.

2. The **evaporator** of the heat pump, in which low-temperature energy generated by the BHE is transferred to the refrigerant of the heat pump and causes a phase change of the refrigerant from liquid to gaseous state.

3. The **compressor** of the heat pump, for compression of refrigerant vapour and to increase its temperature to a level suitable for use in heating systems.

The thermodynamic characteristics of each of these components of the VGS are introduced below.
The typical seasonal variation in the undisturbed temperature $T_0(x,t)$ in the upper soil layer for $x \geq 0$, where x is the depth and t is time, is presented in the right-hand part of Figure 1. An intrinsic feature of the temperature profile in the ground is the presence of a seasonal variability layer of length $L_1 \approx 10\ m$, which results from the interaction between the geothermal heat flux from the Earth, the radiation heat flux from the sun and the molecular heat flux from the atmosphere. The temperature difference on the ground surface $x = 0$ in the mid-latitudes can reach 40°C. The temperature at depth $x \geq L_1$ remains constant, with a low positive gradient in the order of 3°C/100 m. During the cold (heating) season, the temperature in the upper layer is lower than the undisturbed temperature at depth $x \geq L_1$. Hence, the seasonal variability layer reduces the amount of energy extracted by the BHE. The same is true for the summer period, when the VGS is used as a passive thermal source for cooling. Therefore, to increase the energy efficiency of the VGS, it is necessary to exclude the upper layer from energy exchange. Technically, this means thermal insulation of the BHE within this layer. Consequently, the BHE is assumed to be within the layer $x \geq L_1$ with constant temperature T_0.

Based on the proposed schematic diagram of a VGS, it is natural to introduce its thermodynamic scheme in which the low-temperature energy of soil is converted into the high-temperature energy of the compressed refrigerant vapour in the VGS, as shown in Figure 2.

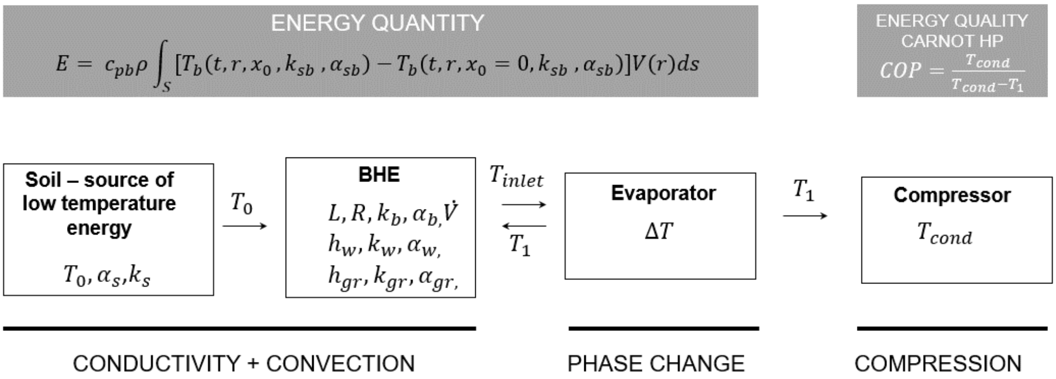

Figure 2. Thermodynamic scheme for a vertical geothermal system (VGS).

Let us now consider the thermodynamic characteristics of each component in the scheme:

- The **energy source (soil)** is characterised by undisturbed temperature T_0 and molecular thermal conductivity and diffusivity k_s and α_s, respectively, and is assumed to be semi-infinite. In soil, heat is transferred by conduction (ground water flow is not considered as well as possible inhomogeneity of thermal conductivity and diffusivity of soil. It is possible to take into account the latter using the dependence of the thermodynamic properties of soils on the depth.)

- The **BHE** is a pipe of radius R and length L in which a secondary fluid with volume flow rate $\dot{V}$, thermal conductivity k_b and diffusivity α_b circulates. Thermodynamic properties of the pipe wall and grout are characterised by the values of their molecular thermal conductivity and diffusivity k_w, k_{gr} and α_w, α_{gr} respectively. The wall thickness of pipe and grout are h_w and h_{gr}, respectively. Heat is transferred in the BHE by conduction and convection.

In the scheme, the heat pump consists of:

- an **evaporator,** the thermodynamic characteristics of which are the temperature difference of brine between its inlet and outlet and the temperature T_1 of the refrigerant vapour at the inlet of the condenser.

- a **compressor** with temperature T_{cond}.

Heat transfer occurs due to phase change in the evaporator and to compression of the refrigerant vapour in the condenser. Thermal, mechanical and electrical losses are not considered here.
The conversion of the low-temperature energy of soil with temperature T_0 into the high-temperature energy of refrigerant vapour with temperature T_{cond} in the scheme consists of two stages:

1 A low-temperature stage during which geothermal energy is extracted from soil into the secondary fluid, which transfers this energy to the refrigerant in the HP evaporator, causing its phase change into vapour.

2 A high-temperature stage during which the refrigerant vapour is compressed and its temperature is increased to the level T_{cond} suitable for heating purposes.

This thermodynamic scheme for VGS does not include the heat distribution circuit (water circuit) and the part of the heat pump refrigeration cycle associated with the condenser and expansion valve. The emphasis is on the laws governing generation of the required amount of energy, which is determined by the interaction between the secondary fluid in the BHE and the refrigerant in the evaporator and by the efficiency of its transformation in the HP, which is characterised by its coefficient of performance (COP). The thermodynamic scheme naturally establishes a quantitative relationship between the amount of energy generated in the evaporator and its quality, characterised by the temperature T_{cond} or COP.

The quantitative and qualitative energy characteristics of the VGS are described below.
As mentioned above, the VGS is a highly nonequilibrium thermodynamic system with different energy transfer mechanisms. For such a system, it is important to establish the relationship between the energy characteristics of the heat pump determined by phase changes of the refrigerant and the characteristics of the BHE with its molecular-

convective energy transfer mechanism. It is obvious that the thermal capacity generated in the evaporator of the heat pump is equal to the difference in enthalpy E_0 of the secondary fluid (brine) at the inlet and outlet of the evaporator, multiplied by its volume flow rate:

$$E_0 = c_p \rho \, \Delta T \, \dot{V} \tag{1}$$

where c_p and ρ are the specific heat capacity at constant pressure and density of the brine in the heat exchanger, ΔT is the temperature difference between the inlet and outlet of the HP evaporator and $\dot{V}$ is the volume flow rate of the brine in the heat exchanger (m^3/s). The magnitude of ΔT is obviously determined by the heat exchanger length and design features of the HP evaporator. Hence, a given capacity of the HP can be provided by finding the length L_0 of the heat exchanger that provides a given value of ΔT at a certain flow rate $\dot{V}$ of brine. Note that equation (1) does not depend on whether the flow of the secondary fluid in BHE is laminar or turbulent and describes a general relationship between the thermodynamic and hydrodynamic parameters of the VGS. It is natural for the flow regime to influence the temperature at the inlet of the HP evaporator.

Equation (1) allows determination of E_{Max}, the maximum thermal capacity generated in the evaporator of a particular HP, *i.e.* for given values of T_0, T_1 (or ΔT) and $\dot{V}$ as:

$$E_{Max} = c_p \rho \, (T_0 - T_1) \, \dot{V} \tag{2}$$

The physical meaning of equation (2) is quite clear; the inlet temperature in the evaporator cannot exceed the value of the unperturbed temperature T_0 of the heat source (soil). The value of the inlet evaporator temperature may be determined by, among other factors, the value of $\dot{V}$, decreasing with increasing $\dot{V}$ and *vice versa*.

In the case of a U-pipe BHE, the statement and solution to the problem of determining the length L_0 depending on the capacity E_0 of the heat pump involve certain difficulties associated with the complicated geometry of the system. Besides, the system is essentially three-dimensional.

A number of studies have devised different simplified models of a U-tube BHE. One example proposes modelling the U-tube BHE by a single pipe surrounded by a concentric ring of grout within a borehole [24-26]. Simplification is achieved by introduction of an equivalent radius of the single pipe. In another U-tube BHE model [27], the borehole is divided into two parts by a vertical plane coming through its centre line and an equivalent radius is introduced for each half-pipe. These models are suitable for studying transient heat transfer in U-tube BHEs.

In this study, we used the U-tube BHE we first proposed in [28]. While the stationary model in that study neglected the influence of pipe wall and grout on the heat exchange between the soil and secondary fluid, in the non-stationary model of VGS their influence can be substantial. Obviously, in the proposed scheme, VGS is characterised by three time scales: $\tau_b = \frac{R^2}{\alpha_b}$, $\tau_w = \frac{h_w^2}{\alpha_w}$ and $\tau_{gr} = \frac{h_{gr}^2}{\alpha_{gr}}$. These scales represent the time to establish thermal equilibrium in the respective medium (brine (b), pipe wall (w), grout (gr)). Soil does not have its own time scale, since it is assumed to be semi-infinite. In this study we considered the case when τ_w, $\tau_{gr} << \tau_b$. Mathematically, this means that the unsteady processes in a VGS are determined by the thermodynamic properties of brine and pipe radius. Estimates of the respective time scales for real operating conditions are provided below. With these assumptions, our U-tube BHE model [28] can be used.

Since the heat exchanger is placed in a medium over which the undisturbed temperature T_0 is uniformly distributed, we assumed that the downward and upward sections of the heat exchanger are thermodynamically decoupled. It is clear that the energy interaction between these downward and upward sections reduces the efficiency of the BHE. Mathematically, this means that D»R, D – distance between legs of BHE. Physically, this assumption means that this model of BHE defines the maximum capacity of the energy well with length L_0 and radius R. For real length scaling it is necessary to introduce a ratio factor that depends on correlation of D and R, length L_0 and thermodynamic properties of soil, well and secondary fluid. Under the decoupling assumption, this interaction is absent and the results represent an upper-bound estimate of the thermal capacity of the U-tube BHE. With this assumption, the extraction of geothermal energy by a U-tube heat exchanger of length L_0 can be represented as heat exchange between the brine flow in a pipe of radius R and length $2L_0$ and the external semi-infinite medium of temperature T_0. Such a heat exchange process is schematically illustrated in Figure 3.

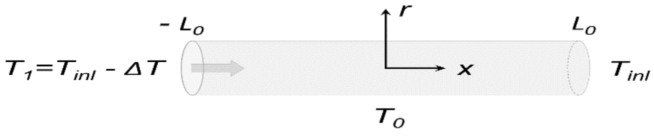

Figure 3. Schematic diagram of an 'unbent' ground U-pipe heat exchanger.

Points $x = -L_0$ and $x = L_0$ in Figure 3 are the outlet and inlet of the HP evaporator (*i.e.* the temperature at these points is equal to the temperature of the brine at the outlet and inlet of the evaporator).

3. Governing equations

We chose to use cylindrical coordinates (r, x) with origin at the middle of the heat exchanger. In this coordinate system, the heat exchange between the brine and the soil is described by the following system of equations:

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial T_s}{\partial r}\right), \quad \text{for } r \succ R \tag{3}$$

$$\frac{\partial T_b}{\partial t} + V_x \frac{\partial T_b}{\partial x} = \alpha_b \left(\frac{\partial^2 T_b}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial T_b}{\partial r}\right) \text{ for } r < R \tag{4}$$

where the subscripts s and b refer to the soil and the brine, respectively, and α_b and α_s are the thermal diffusivity of the brine and soil, respectively. In Eq. (4), it is assumed that convective heat transfer in brine occurs in the longitudinal direction only and therefore it can be assumed to be Poiseuille flow:

$$V_x = V_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{5}$$

where V_{max} is the maximum velocity of Poiseuille flow on the pipe axis (r = 0). The assumption that the brine flow is laminar is unessential, but considerably simplifies the solution of the problem and the interpretation of results. In addition, we assumed that the effect of the pipe wall and grout on the heat exchange between the soil and the brine can be neglected. This assumption is also unessential, because the model describes the energy interaction between the energy source (soil) of infinite spatial scale and the energy carrier (secondary fluid) of finite spatial scale. However, it may appear very important to take into account the effect of pipe wall and grout in analysing the transient characteristics of BHE and this is considered below.

4. Dimensionless variables

The system of equations (3), (4) in dimensionless variables has the form:

$$\alpha_{sb} \frac{\partial \theta_s}{\partial t} = \frac{\partial}{\partial r} r \frac{\partial \theta_s}{\partial r}, \text{ for } r \succ 1 \tag{6}$$

$$\frac{\partial \theta_b}{\partial t} + (1 - r^2)\frac{\partial \theta_b}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \theta_b}{\partial r}, \text{ for } r \prec 1 \tag{7}$$

The dimensionless parameters in Eqs. (6)-(7) are defined as:

$$r^* = r/R, \quad x^* = x/L_x, \; t^* = tR^2/\alpha_b \tag{8}$$

$$\theta_{s,b}(t, r, x) = \frac{T_{s,b} - T_1}{T_0 - T_1} \tag{9}$$

Since only dimensionless variables are used below, asterisks in Eqs. (6), (7) are omitted for convenience and $\alpha_{sb}= \alpha_s/\ \alpha_b,\ L_x = \frac{2E_{max}}{\pi k_b(T_0 - T_1)}$.

5. Initial and boundary conditions

The next step was to formulate the boundary conditions for Eqs. (6), (7). Accepting the conditions of continuity for temperature and heat fluxes on the wall of borehole, then:

$$\theta_s = \theta_b,\ \frac{\partial\theta_b}{\partial r} = \frac{k_s}{k_b}\frac{\partial\theta_s}{\partial r} \quad \text{for } r = 1, \text{ and all t and x,} \tag{10}$$

initial conditions

$$\theta_s = \theta_b = 1 \text{ at t=0 and all r and x} \tag{11}$$

and

$$\theta_s \to 1 \text{ as } r \to \infty, \text{ for all } x \tag{12}$$

Besides, θ_b is finite at $r = 0$.

As Eq. (7) is parabolic, only one boundary condition is required for the longitudinal coordinate x. Therefore, to make the solution of the boundary-value problem more convenient, we placed the origin of coordinates (r, x) at the point outlet of the evaporator of the heat pump (Figure 1). Then Eq. (9) yields:

$$\theta_b = 0, \text{ for } x = 0 \text{ and } r \prec 1 \tag{13}$$

Thus, the system of Eqs (6), (7) with boundary and initial conditions (10)-(13) and the assumptions made above describes the unsteady energy exchange between the soil, the vertical heat exchanger and the heat pump. In such a problem statement, the geothermal system is a single thermodynamic system in which the energy exchange between the soil, the vertical heat exchanger and the heat pump is a coupled process.

Note that the boundary-value problem depends only on two dimensionless similarity parameters $k_{sb} = k_s/k_b$ and $\alpha_{sb}= \alpha_s/\ \alpha_b$, which are the thermal diffusivity and thermal conductivity ratio of soil and brine.

6. Results

The boundary-value problem in Eqs. (6), (7), (10)-(13) was numerically solved for different values of the similarity parameters k_{sb} and α_{sb} (*i.e.* for various types of soil and brine) and the distribution of temperature fields both inside $\Theta_b(t, r, x_0, k_{sb}, \alpha_{sb})$ and outside $\Theta_s(t, r, x_0, k_{sb}, \alpha_{sb})$boreholes was determined. This section presents the simulation results for two characteristic cases: 30% propylene glycol and 70% water as the brine, and sand with 20% water content and granite as two types of soil. Similarity parameters for these cases have numerical values k_{sb}= 3.02, α_{sb}= 8.42 and k_{sb}=7.95, α_{sb}= 10.8, respectively.

The next step was to estimate the characteristic time scales for a typical borehole with PE pipes, R = 2 cm, hw = 2 mm, $\alpha_w = 2.1\text{x}10^{-7}$ m^2/s [29], brine with $\alpha_b = 1.1\text{x}10^{-7}$ m^2/s [30] and bentonite grout with $\alpha_{gr} = 1.9\text{x}10^{-7}$ m^2/s [31]. Assuming $h_{gr} = 10^{-2}$ m, it can easily be determined that $\tau_b \sim 1$ hour, $\tau_w \sim 20$ s and $\tau_{gr} \sim 0.14$ hour. Consequently, the influence of pipe wall and grout on the non-stationary processes in VGS is essential only during initial times ~0.14 hour. At long times, unsteady processes in VGS are determined by thermodynamic properties of secondary fluid and soil.

We also analysed the thermodynamic cycle of the VGS and, in particular, its energy characteristics. It is natural to define the debit (capacity) of the well as:

$$E(t, x_0, \alpha_{sb}, k_{sb}) = c_{pd}\rho_b \int_S [T_b(t, r, x_0, \alpha_{sb}, k_{sb}) - T_b(t, r, x_0 = 0, \alpha_{sb}, k_{sb})] V(r) ds \tag{14}$$

where $T_b(t, r, x_0, \alpha_{sb}, k_{sb})$ is the temperature of the brine at the inlet of the evaporator; $T_b(t, r, x_0 = 0, \alpha_{sb}, k_{sb})$ is the temperature of the brine at the outlet of the evaporator; $V(r)$ is the brine velocity in the longitudinal direction; and S is the cross-sectional area of pipe in the heat exchanger. Equation (14) is the difference in enthalpy of the brine at the inlet and outlet of the HP evaporator. Note, that Eq. (14) is valid for any flow regime of brine. In turn, the flow regime determines the temperature distribution $T_b(t, r, x_0, \alpha_{sb}, k_{sb})$. As already indicated, in this study we considered Poiseuille flow.

The dimensionless debit of the energy well can be defined as:

$$E^*(x_0, \alpha_{sb}, k_{sb}) = \frac{E}{E_{max}} \tag{15}$$

Physically, Eq. (15) is the ratio of the unstationary debit of an energy well of length x_0 to the debit of a well with $\Delta T = T_0 - T_1$, and it is obvious that $E^*(t, x_0, \beta, \gamma) < 1$.
It can easily be deduced that:

$$E^*(t, x_0, \alpha_{sb}, k_{sb}) = 4\int_0^1 \theta_b(t, r, x_0, \alpha_{sb}, k_{sb})(1 - r^2) r dr \tag{16}$$

Figure 4 shows the capacity of a BHE as a function of dimensionless length and dimensionless time for two types of soil. Values of dimensionless time t are indicated above the corresponding curves. The dashed lines correspond to the stationary regime at $t \to \infty$.

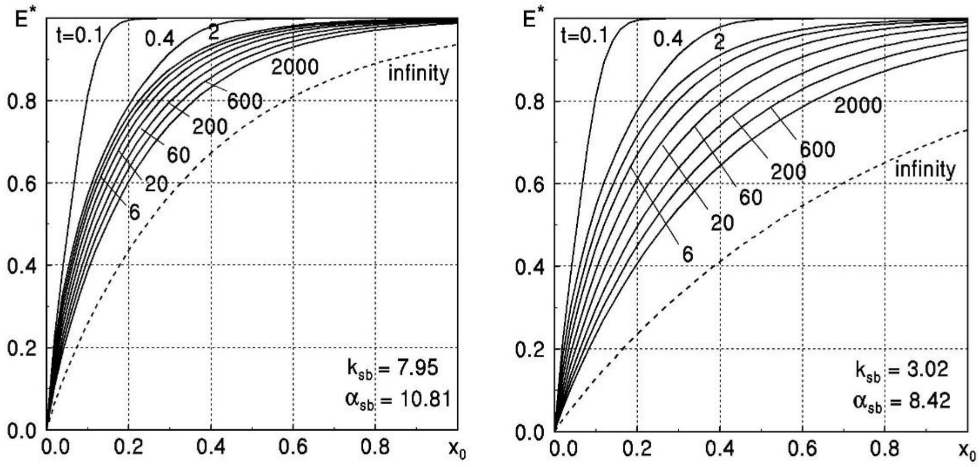

Figure 4. Capacity of a borehole heat exchanger (BHE) as a function of its dimensionless length x_0, dimensionless time t and soil-brine similarity parameters α_{sb}, k_{sb} for (left) granite and (right) sand with 20% water content.

It is obvious that $E^*(t, x_0{=}0, \alpha_{sb}, k_{sb}) = 0$, $E^*(t, x_0, \alpha_{sb}, k_{sb}) \to 1$ as $x_0 \to \infty$ and $E^*(t, x_0, \alpha_{sb}, k_{sb})$ increases with k_{sb} for fixed values of x_0 and t.

Note the strong nonstationarity of changes in BHE capacity at the initial time. The rate of change in BHE capacity $\frac{\partial E^*}{\partial t}$ for two values of x_0 is illustrated in Figure 5.

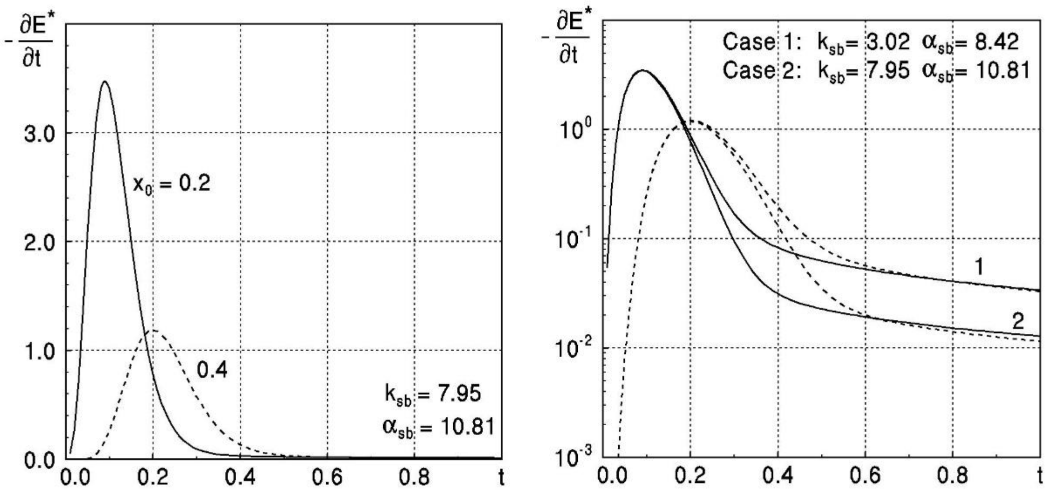

Figure 5. Calculated rate of change in borehole heat exchanger (BHE) capacity $\frac{\partial E^*}{\partial t}$ for (left) dimensionless BHE length $x_0 = 0.2$

(solid line) and $x_0 = 0.4$ (dashed line) and (right) for sand with 20% water content (case 1) and granite (case 2).

The rate of change in BHE capacity has a maximum that depends on the length of x_0 and is achieved at t = 0.1 for $x_0 = 0.2$ ($\frac{\partial E^*}{\partial t}\max = 3.2$) and t = 0.2 for $x_0 = 0.4$ ($\frac{\partial E^*}{\partial t}\max = 1.1$). It should be noted that the value of the maximum and the time at which it is reached do not depend on soil type. The rate $\frac{\partial E^*}{\partial t}$ asymptotically tends to zero with increasing time, but its value depends on the type of soil. It should be remembered that for small times t < 0.2, the impact of grout can be significant. However, as follows from the above graphs, the capacity of BHE becomes quasistationary when t > 0.4. The degree of unsteadiness depends on the type of soil and the length of the BHE, as clearly seen in Figure 4. The results show that at the beginning of VGS operation, the capacity of the energy well is sharply reduced. The physical meaning of this result is quite obvious: at the initial moment the temperature difference between secondary fluid and soil, and heat fluxes are maximal. Therefore, the rate of decrease of the energy well capacity is also maximal. With increasing of time, the heat fluxes are reduced and the capacity of BHE goes to the quasi-stationary state. However, at time t > 0.4 the rate of reduction in capacity decreases to values in the order of 10^{-2}, and with increasing time of operation of the VGS the rate tends to zero. Note also that the stationary functioning mode of an energy well is achieved asymptotically as $t \rightarrow \infty$. Thus, knowing the operating time characteristics of VGS, it is possible to calculate the optimal length of the collector to produce the given amount of energy.

Another issue of energy well functioning is associated with the quality of the energy produced, which is characterised by the value of COP for the VGS and is determined (besides construction features of a HP) by the value of the temperature of the brine at the inlet of the evaporator $T_b(t, r, x_0, \alpha_{sb}, k_{sb})$.

Consider the laws of formation of the temperature of the brine at the inlet of the evaporator. The inlet temperature, averaged over the cross-section of the pipe, is defined by $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb}) = 2\int_0^1 \theta_b(t, x_0, r, \alpha_{sb}, k_{sb}) r dr$ and its value determines the value of the COP. Figure 6 shows the average dimensionless temperature $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ as a function of the dimensionless length x_0 and similarity parameters for different time t.

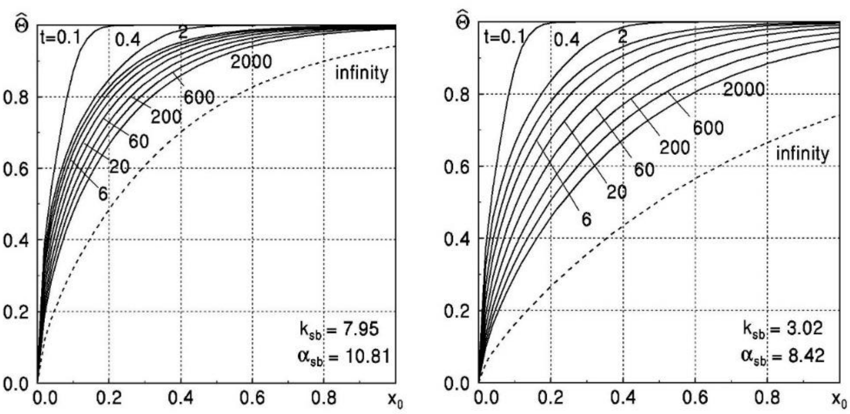

Figure 6. Calculated average dimensionless temperature $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ for different values of borehole heat exchanger (BHE) dimensionless length x_0 and similarity parameters α_{sb}, k_{sb} for different time t in (left) granite and (right) sand with 20% water content.

The values of t are indicated above the corresponding curves. These curves show the same behaviour as the well power $E^*(t, x_0, \alpha_{sb}, k_{sb})$ and monotonically increase with x_0 and k_{sb}. Note that the dimensionless characteristics $E^*(t, x_0, \alpha_{sb}, k_{sb})$ and $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ have absolutely different physical sense: $E^*(t, x_0, \alpha_{sb}, k_{sb})$ is the capacity or the amount of energy per unit time generated by the BHE and $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ is the temperature at the inlet to the evaporator of the heat pump that determines the COP of the VGS. Physically, $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ characterises the quality of extracted heat (the COP increases/decreases with $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$) and the pair $\Omega(t, x_0, \alpha_{sb}, k_{sb}) = \{E^*(t, x_0, \alpha_{sb}, k_{sb}), \hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})\}$ defines the energy characteristics of the energy well or BHE. It is clear that an increase in the length of a single well increases the amount of energy and improves its quality (COP). If, however, there are several wells, the relationship between the amount and quality of energy extracted is more complex. Several parallel wells of total length L_0 in a highly conductive medium ($k_{sb} > 1$) will produce much more energy than a single well of the same length L_0. However, it is obvious that the COP of the system of wells is lower than the COP of a single well.

Thus in summary, the combined numerical/analytical relationship $\Omega(t, x_0, \alpha_{sb}, k_{sb}) = \{E^*(t, x_0, \alpha_{sb}, k_{sb}), \hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})\}$ completely defines the energy characteristics of an energy well for a given VGS and has three parameters: x_0, the dimensionless length of the BHE (quantitative characteristic), and α_{sb} and k_{sb}, which characterise the thermal properties of the brine-ground/soil system. Note that the function $\Omega(t, x_0, \alpha_{sb}, k_{sb})$ for VGS is fundamental and independent of the characteristics of the heat pump.

These results can be used as a basis for designing the optimal energy well, which is an energy well of a given capacity generating energy of the required quality when the BHE has minimum total length. The quality of the energy generated by VGS can naturally be characterised by the COP. It is clear that meaningful values on the optimal energy well for VGS can be obtained when $\Omega(t, x_0, \alpha_{sb}, k_{sb})$ varies nonlinearly with x_0 and t.

7. Conclusions

This paper presents an unsteady phenomenological hydrothermodynamic model of a VGS, considered as combining soil, BHE and GSHP in an integrated nonequilibrium thermodynamic system and taking into account both spatial and temporal variability of ground/soil temperature and the energy characteristics of the heat pump.

This VGS model disregards mechanical, electrical and hydraulic losses. Thermodynamically, the components of the VGS are characterised by essentially different energy transfer mechanisms and are related by continuity conditions for temperature and heat fluxes. The scheme developed for VGS naturally establishes a quantitative relationship between the amount of the energy generated in the evaporator and its quality characterised by the inlet temperature $\hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})$ or COP. A special class of nonequilibrium space-time scales defined by the energy characteristics VGS is introduced.

The present analysis showed that with appropriately chosen variables, all energy characteristics of VGS are determined by two similarity parameters: α_{sb}, the thermal diffusivity ratio of soil and brine, and k_{sb}, the thermal conductivity ratio of soil and brine. The concept of debit or capacity of the energy well was introduced and its dependence on the time and well length was analysed. The model developed makes it possible to simulate transient operating conditions of VGS. In particular, it was shown that the unsteadiness of functioning of BHE is characterised by three time scales (of brine, pipe wall and grout): $\tau_b = \frac{R^2}{\alpha_b}$, $\tau_w = \frac{h_w^2}{\alpha_w}$ and $\tau_{gr} = \frac{h_{gr}^2}{\alpha_{gr}}$. As examples, changes in the BHE capacity at constant operating conditions of HP were calculated for two types of soil. The results showed that at the beginning of VGS operation the capacity of an energy well is sharply reduced. However, at time t > 0.4 the rate of reduction in capacity of the BHE decreases to values in the order of 10^{-2} and with increasing time of operation of the VGS the rate tends to zero. The rate of change in BHE capacity has a maximum that depends on its dimensionless length x_0 and is reached at t = 0.1 for x_0 = 0.2 ($\frac{\partial E^*}{\partial t}\max = 3.2$) and t = 0.2 for x_0 = 0.4 ($\frac{\partial E^*}{\partial t}\max = 1.1$). The values of the maximum and the time at which it is reached do not depend on soil type. The rate $\frac{\partial E^*}{\partial t}$ asymptotically tends to zero with increasing time, but its value depends on the type of soil. The stationary energy well functioning mode is achieved asymptotically as $t \to \infty$. Thus, knowing the operating time of the heat pump cycle, it is possible to calculate the optimal length of the collector to produce the given amount of energy.

The fundamental energy characteristic $\Omega(t, x_0, \alpha_{sb}, k_{sb}) = \{E^*(t, x_0, \alpha_{sb}, k_{sb}), \hat{\theta}(t, x_0, \alpha_{sb}, k_{sb})\}$ of an energy well for a given VGS was introduced. It was shown that $\Omega(t, x_0, \alpha_{sb}, k_{sb})$ completely defines the quantitative and qualitative energy characteristics of a given VGS and has three parameters: x_0, the dimensionless length of BHE (quantitative characteristic), and α_{sb}, k_{sb}, which characterise the thermal properties of the ground/soil and brine (α_{sb} defines the time scale of the problem and k_{sb} determines the space scale).

The results obtained enable innovative strategies for future VGS design to be developed, i.e. how to design an optimal VGS that generates high temperature energy of required amount and quality with minimal length of BHE, using a given operating time of the heat pump cycle.

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