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Improving analytical solutions for energy pile/borehole with groundwater flow

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Abstract

Mean integral solutions to the equations of the moving finite line- source (MFLS) and the moving spiral source (MFSS) models for ground heat exchangers with groundwater flow are presented. The solutions for the average temperature in the MFLS and MFSS models are expressed, respectively, as single and double integrals averaging the exact solutions over both depth and circumference. Depth to diameter ratio of vertical heat source defines the intermediate time interval, where thermal capacities of energy pile/borehole and flowing groundwater influence significantly on temperature response. Approximate expressions for the average temperature are derived analytically from the MFLS model within the intermediate time scale.

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1. Introduction

A ground coupled heat pumps (GCHP) is one of the sustainable energy technologies for heating and cooling buildings. GCHPs are based on borehole heat exchangers (BHEs) consisting of a U-loop tube buried in the ground and connected to a heat pump through which a heat carrier fluid is circulated.

GCHP systems provide renewable energy to buildings, though a long-term character of the financial benefit and space requirements prevent their widespread. One possible method of reducing the space requirement is to place tubes of heat exchangers in foundation piles beneath buildings. Recently, such innovated GCHP systems in foundation piles have also attracted interest of geotechnical engineers. Figure 1 represents GCHP systems with vertical U-loops or spiral coils integrated in energy piles, which provide both the structural support and cooling/heating buildings. Ground coupled heat exchangers (GHE) are an alternative to more expensive ground heat exchangers used in ground-source heat pump systems for nearby houses.

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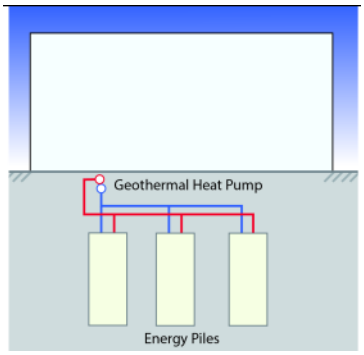


Fig. 1. Diagram of geothermal U-loop inserted inside the borehole.

To increase cost efficiency and to improve the GHE performance a number of simulation models and methods are used for its proper sizing. Some software programs are based on the cylindrical or line source models [1] that account for the conduction heat transfer mechanism in the surroundings [2-8]. However, for the groundwater that may be in the geologic formations the convective heat transfer cannot be neglected. These convection effects have been taken into account, using the moving source method (Carslaw and Jaeger, 1959) for a number of models for vertical GHE ranging from line [9-14] to spiral [15, 16] sources of heat, including solid cylinder one [17].

Within the frame of the line-source models, the so-called "***g***-function" introduced by Eskilson (1987), which represents the thermal response factor of the borehole to a heat pulse, is used when modeling the three-dimensional (3D) temperature distribution of a multiple borehole configuration [6]. In this basic case, thermal response is proportional to the thermal resistance of the ground (Sutton et.al, 2003). ***G***-functions, accounting also for a thermal resistance of the GHE interior, have been introduced [7] and developed [8] within the framework of the solid cylinder-source model [4, 20], generalizing the ***g***-function. Actually, there is a variety of thermal response functions: some of them describe the shallow GHE modeled as cylinder or spiral heat sources, account for the inner thermal resistance of the energy piles and thus improve in this respect the thermal response functions from the line-source model, which neglect this.

Infinite line-source models without groundflow have some limitations: the approximation to the ground temperature does not reach a steady-state value. This is improved in the two dimensional infinite line-source model for the BHEs, that takes into account the prevailing groundwater: its solution does have a steady-state limit [9]. This is also the case, for the finite line-source (FLS) model: the approximation to the ground temperature does reach a steady-state value.

Some line-sources models account for a thermal resistance of the ground in the GHE surroundings. In this respect it is important to notice that different properties of the ground can produce opposite influence on the evolution of the temperature distribution: for instance, accounting for an anisotropy of the ground, as calculated in the FLS model, normally results in increasing the time of steady-state attainment [18], while as it is shown in the moving infinite line-source model, the presence of groundwater flow decreases it [9, 10].

In general, regardless of which heat source geometry is selected, ***z***-dependent solution for the combined heat transfer is currently used in geothermal applications to estimate the thermal response with groundwater flow.

Further refinement of the thermal response functions in the moving finite line-source (MFLS) and in the moving finite spiral-source (MFSS) models is desirable, in particular, since numerical calculations show that the evaluation at the mid-point of the borehole overestimates the steady-state temperature. This study considers the combined effect of convection and thermal conduction for the mean temperature response of a GHE, modeled as line and spiral source with groundwater flow, which is parallel to the surface of the horizontally stratified ground. Effect of the thermal storage capacity of the energy pile can be analysed by the mean lumped moving finite spiral source method.

	Nomenclature
D	Distance from the ground surface to the thermo-active part of the borehole [m]
$\hat{g}$	Extended thermal response function, [-]
H	Depth of the GHE [m]
k	Wave number unit [1//m]
r	Radial coordinate [m]
r_0	Radius of the ground heat exchanger [m]
q_z	Heat flow per length unit [W/m]
t	Time [s]
$t_r = r_0^2/\alpha$	Short time scale for the borehole heat exchanger [s]
$t_z = H^2/\alpha$	Large time scale for the borehole heat exchanger [s]
T	Temperature [K or ºC]
T_0	Undisturbed ground temperature [K or ºC]
U	Effective groundwater flow velocity, [m/s]
z	Vertical axial coordinate [m]
Greek Letters	
$\alpha = \lambda/C$	Effective ground thermal diffusivity, [m^2/s]
γ	Euler's constant
λ	Effective ground thermal conductivity, [W/K/m]
Superscripts	
$<...>_z \ (= \frac{1}{H}\int_D^{H+D} ... \, dz)$	Integral mean over axial co-ordinate
$<...>_\varphi \ (= \frac{1}{2\pi}\int_0^{2\pi} ... \, d\varphi)$	Integral mean over azimuthal co-ordinate
Subrscripts	
MFLS	Moving finite line-source model
MFSS	Moving finite spiral-source model

The rest of the paper is organized as follows. Section 2 reviews the classical moving infinite line-source (MILS) theory, introduces the MFLS standard model formulation, presents the new solutions, and highlights the way to overcome the limitations of the MILS model. The proposed method based on the integral mean temperature to take account consistently for the vertical heat transfer effects is then applied to the solution from moving finite spiral-source model to obtain new mean solution. The findings are then analysed theoretically and justified through the comparison with the classical solutions. Finally, Section 3 gives some conclusions.

2. Some models for vertical GHE with groundwater flow.

Typically, in the GCHP applications, one applies the ***g***-function introduced by Eskilson (1987), which represents the temperature response from line-source model of the GHE at a borehole radius r_b to a heat pulse at the mid-depth, i.e. at ***z=H/2***. Here we define the extended thermal response function as follows

$$T(z,r,\varphi;t) - T_0 = \frac{q_z}{2\pi\lambda}\hat{g}(z,r,\varphi;t), \qquad (1)$$

so that, $\hat{g}$ is the dimensionaless temperature at a given point of space. The extended $\hat{g}$ -function is reduced to the ***g***-function at specific values of the co-ordinates ***z*** and ***r*** : $g = \hat{g}(z = H/2, r = r_b, \varphi; t)$. When heat is released at a constant rate along the BHE, and is transferred by thermal conduction and convection, the solutions to the

problem can be expressed through the $\hat{g}$ -function for the both line- and spiral-source models. The variety of thermal response functions includes ones developed for different boundary conditions for geothermal bore fields [6, 20].

To determine the thermal resistance, taking into account the asymetric temperature distribution with respect to the direction of the ground flow, Sutton et al. (2003) recommended to use the mean temperature averaging along the GHE perimeter, i.e. over the angular variable φ ,

$$\left\langle \hat{g}(z,r;t)\right\rangle_{\varphi} = \frac{1}{2\pi}\int_{0}^{2\pi} \hat{g}(z,r,\varphi;t)d\varphi, \tag{2}$$

For a constant groundwater velocity, the solution for temperature from a buried heat source is obtained in a coordinate system co-moving with the groundwater in the same direction, $\boldsymbol{U}=\{U,0,0\}$ [1].

For sufficiently large times, in the frame of the line-source theory for the borehole, the MILS model gives thermal resistance of ground at a borehole radius $\boldsymbol{r_b}$ in the following form (Sutton et al., 2003):

$$R_{\substack{ground\\transient\\convection}} = \frac{1}{2\pi\lambda}\left\langle \hat{g}_{MILS}\left(r=r_b,t\right)\right\rangle_{\varphi} \quad , \quad \left\langle \hat{g}_{MILS}\left(r,t\right)\right\rangle_{\varphi} = \frac{1}{2} I_0\left(\frac{rU}{2\alpha}\right)\Gamma\left(0, {r^2}/{4\alpha t}; \left(\frac{rU}{4\alpha}\right)^2\right) \tag{3}$$

Here function $\Gamma\left(\mathrm{a},z;b\right)$ denotes the ‘‘generalized incomplete gamma function’’ introduced by Chaudhry and Zubair (1994),

$$\Gamma(a,z;b) = \int_{z}^{\infty} t^{a-1}e^{-t-b/t}dt = 2\int_{\sqrt{z}}^{\infty} t^{2a-1}e^{-t^2-b/t^2}dt \tag{4}$$

and $\boldsymbol{U}$ is the effective groundwater flow velocity for a porous soil modeled as a saturated porous matrix (Sutton et al., 2003). Note that in the steady–state conditions, $\Gamma\left(0,0;b\right) = 2K_0\left(\sqrt{4b}\right)$ is just the modified Bessel function of the second kind of order zero, so that the classical solution (Carslaw and Jaeger, 1959; Sutton et al., 2003) is recovered in the following form

$$R_{\substack{steady\\convection}} = {1}/{2\pi\lambda}\left\langle \hat{g}_{MILS}\left(r=r_b,t\to\infty\right)\right\rangle_{\varphi}, \qquad \left\langle \hat{g}_{MILS}\left(t\to\infty\right)\right\rangle_{\varphi} = I_0\left(\frac{rU}{2\alpha}\right)K_0\left(\frac{rU}{2\alpha}\right) \tag{5}$$

Note that in the generic case: $\boldsymbol{U}=\{u_x, u_y,, u_z\}$ the above solution remains the same, but with $U=\sqrt{{u_x}^2+{u_y}^2}$, therefore, the u_z effects are beyond the scope of the infinite heat source model. This is in contrast to the effect of the vertical component of groundwater seepage on thermal behavior of the energy pile predicted by the finite line-source model (Zhang et al., 2017).

This paper considers heat flow along the vertical *z*-axis with a constant groundwater velocity in a radial direction using the mean integral method for heat source of finite depth in the semi-infinite region.

2.1. New mean solution for the case of buried finite line-source with groundwater

In order to estimate the thermal resistance at the wall of a GHE, the mean temperature around a circle is to be estimated for the moving line-source of finite depth, characterized by top distant $\boldsymbol{D}$ from the ground surface, (Fig.2a) in the same way as it was done for the MILS model in [9, 10].

First, the temperature response from the moving finite line-source (MFLS) model is averaged, integrating the exact solution along the BHE circumference, and then dividing by 2π **,** Eq.(2). This $\boldsymbol{z}$-dependent solution for the temperature response proposed for $\boldsymbol{D=0}$ by Molina-Giraldo et al. (2011) was presented in an alternative

equivalent form by Zhang et al. (2016).

The *new* azimuthal mean temperature response for arbitrary $\boldsymbol{D}$ can be defined for the ground flow in the $\boldsymbol{x}$-direction as follows:

$$\left\langle \hat{g}_{MFLS}\left(H,D,z,r;t\right)\right\rangle_{\varphi}=\frac{1}{2}I_0\left(\frac{Ur}{2\alpha}\right)\int\limits_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}\Phi(Hu,Du,zu)\cdot\exp\left(-\left(\frac{U}{4\alpha u}\right)^2-u^2r^2\right)$$

$$\Phi(h,d,z)=\frac{2}{\sqrt{\pi}}\int_d^{h+d}\left[e^{-(z-z')^2}-e^{-(z+z')^2}\right]dz' \tag{6}$$

$$\Phi(h,d,z)=-erf\left(d-z\right)+erf\left(h+d-z\right)+erf\left(d+z\right)-erf\left(h+d+z\right),$$

where $\boldsymbol{I_0}$ denotes the modified Bessel function of the zero-th order, $\left\langle \hat{g}_{MFLS}\left(H,D,z,r;t\right)\right\rangle_{\varphi}$ is the dimensionless temperature at $\boldsymbol{z}$ point and radial distance $\boldsymbol{r}$ from the axis of the finite line-source (FLS) of heat starting at $\boldsymbol{z}$= $\boldsymbol{D}$ from the ground surface.

Second, the temperature response is averaged, integrating the exact solution for a constant heat rate along the BHE, and then dividing by $\boldsymbol{H}$, i.e. the mean temperature for the buried GHE is defined as follows

$$\left\langle \hat{g}_{MFLS}\left(H,D,r;t\right)\right\rangle_{z,\varphi}=\frac{1}{H}\int\limits_{D}^{H+D}\left\langle \hat{g}_{MFLS}\left(H,D,z,r;t\right)\right\rangle_{\varphi}dz, \tag{7}$$

Because of the elementary form of the double integral representation for the mean solution, we reduce it to the single integral

$$\left\langle \hat{g}_{MFLS}\left(H,D,r;t\right)\right\rangle_{z,\varphi}=\frac{1}{2}I_0\left(\frac{Ur}{2\alpha}\right)\int\limits_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}S_D(Hu,Du)\exp\left(-\left(\frac{U}{2\alpha u}\right)^2-u^2r^2\right) \tag{8}$$

$$S_D\left(h,d\right)=\frac{1}{h}\int_d^{h+d}dz'\,\Phi(h,d,z')=\frac{2}{h\sqrt{\pi}}\int_d^{h+d}\int_d^{h+d}\left[e^{-(z-z')^2}-e^{-(z+z')^2}\right]dzdz'$$

The above double integral can be expressed as [20]

$$S_D(h,d)=L(h)-L(2d)D/H-L(2d+2h)(D/H+1)+L(2d+h)(2D/H+1) \tag{9}$$

$$L(h)=2\cdot erf\,(h)-2\frac{1-e^{-h^2}}{h\sqrt{\pi}}=ierf\,(h)\Big/2h$$

where ***ierf (h)*** is the integral error function [1]. In addition, use of the single-integral form of the solution for the mean temperature results in a drastic decrease of the CPU time when compared to the double integral.

2.2. Approximations of the mean solution from moving finite line-source: from finite (MFLS) to infinite (MILS)

Since $\boldsymbol{S_D \longrightarrow 2}$, as $\boldsymbol{H \longrightarrow \infty}$ in Eq.(9), the mean MFLS solution recovers the solution for the infinite line-source MILS model in Eq.(3) (Sutton et. al, 2003), thus justifying our solution $\left\langle \hat{g}_{MFLS}\left(H,D,r;t\right)\right\rangle_{z,\varphi}$ in (8). Furthermore, we derived the asymptotic expansion of the integral solution for small values of the parameters *r/H*, $\sqrt{4\alpha t}/H$ for a GHE starting from the ground surface at the $\boldsymbol{z}$=0,

$$\left\langle \hat{g}_{MFLS}\left(H,r,U;t\right)\right\rangle_{z,\varphi}\ \hat{g}_{MILS}\left(r,U;t\right)-I_0\left(2\sqrt{b}\right)\cdot\frac{3r}{4H\sqrt{\pi}}\Gamma\left(-\frac{1}{2},x^2;b\right)+O\left(\frac{r}{H}\right)^2,\quad t<<\frac{H^2}{4\alpha} \tag{10}$$

$$b=\left(\frac{rU}{4\alpha}\right)^2, \quad x=r/\sqrt{4\alpha t} \tag{11}$$

The above equation contains generalized incomplete gamma function, Eq. (4), for ***a=-1/2.*** This can be expressed in terms of exponential and complementary error functions as follows [20]

$$\Gamma\left(-\frac{1}{2},x^2;b\right)=\frac{\sqrt{\pi}}{2\cdot\sqrt{b}}\left[erfc\left(-\frac{\sqrt{b}}{x}+x\right)\exp\left(-2\sqrt{b}\right)-erfc\left(\frac{\sqrt{b}}{x}+x\right)\exp\left(2\sqrt{b}\right)\right], \tag{12}$$

and, therefore, the solution is also expressed in the terms of these functions. Above result clarifies the roles played by the finite-depth corrections in the heat exchange with the surrounding ground for the time scales.

In addition, we derived the expansion of the above one for small values of the both parameters $r/\sqrt{4\alpha t}$ and $4b/x^2=U^2t/\alpha$ **,** i.e. $r^2/\alpha \ll t \ll \alpha/U^2$, then the series in these terms can be presented as

$$\left\langle \hat{g}_{MFLS}\right\rangle_{z,\varphi} \approx \frac{1}{2}I_0\left(2\sqrt{b}\right)\cdot\left[\left(-\ln\frac{r^2}{4\alpha t}-\gamma\right)(1+b)-\frac{U^2t}{4\alpha}+b-\frac{3r}{H\sqrt{\pi}}\left(-\sqrt{\pi}\left(1+\frac{2b}{3}\right)+(1+b)/x+\left(1+\frac{b}{2}\right)x-\frac{b}{3x^3}\right)\right] \tag{13}$$

where ***b, x*** are defined in Eq.(11). This expansion is valid for small groundwater velocity values in the interval $U \approx \alpha/r$.

Note that, in the limiting case of ***U=0***, the proposed expression (13) at ***I_0(0)=0*** recovers the known thermal response function in the FLS model (Bandos et.al, 2007). Furthermore, the groundwater velocity and finite-size corrections in Eq. (13) give a lower value for the thermal response than that predicted by the MILS model.

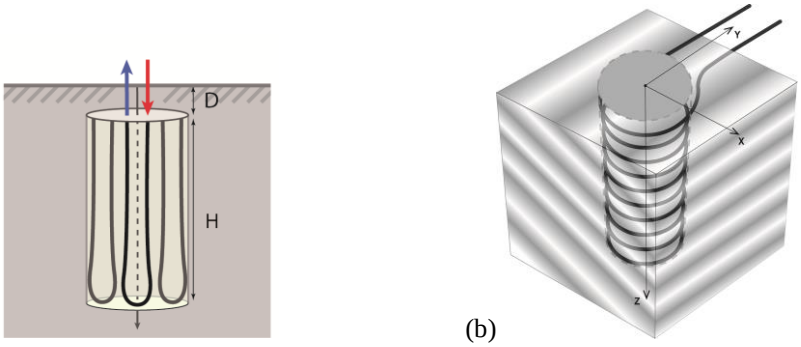
New approximate expressions are proposed for the mean dimensionless ground temperature around a deep GHE (***H/r>>1***) for the intermediate-time intervals, i.e. $r^2/\alpha \ll t \ll \alpha/U^2$ and $t \ll \frac{H^2}{4\alpha}$ for the combined heat conduction and convection problem. There are two characteristic time scales: $t_z=\frac{H^2}{4\alpha}$ and $t_u=\frac{\alpha}{U^2}$ that govern the process of approaching the steady-state. This can be seen from the general solution for ***D=0***,

$$\begin{aligned}&\left\langle \hat{g}_{MFLS}\left(H,r,U;t\right)\right\rangle_{z,\varphi}=\frac{1}{2}I_0\left(\frac{Ur}{2\alpha}\right)\int_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}\left[2\cdot L\left(Hu\right)-L(2Hu)\right]\exp\left(-\frac{b}{u^2r^2}-u^2r^2\right),\\&\int_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}\cdot L\left(Hu\right)\cdot\exp\left(-\left(\frac{U}{4\alpha u}\right)^2-u^2r^2\right)=2\cdot\int_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}\operatorname{erf}(Hu)\exp\left(-\frac{b}{u^2r^2}-u^2r^2\right)-\frac{r}{H}\Gamma\left(-\frac{1}{2},\frac{r^2}{4\alpha t};\frac{r^2}{4\alpha^2}U^2\right)\\&+\frac{\sqrt{r^2+H^2}}{H}\Gamma\left(-\frac{1}{2},\frac{r^2+H^2}{4\alpha t};\frac{r^2+H^2}{4\alpha^2}U^2\right)\end{aligned} \tag{14}$$

obtained from Eqs.(8) and (9).

This is relevant when determining the characteristic time of reaching the steady-state temperature distribution around the MFLS of heat affected by the groundwater flow. In the following section the temperature of a vertical GHE with spiral heat carrier tubes is assessed using the same approach.

2.3. New mean solution from moving finite spiral source of heat



(a) (b)

Fig. 2. Scheme of thermo-active foundations with vertical U-tube (a) and spiral (b) type GCHP

The three-dimensional temperature distribution in semi-infinite medium with spiral heat generation sources was presented in [16]. This temperature in a form of the double integrals is averaged, integrating this solution around and along the GHE. We reduce it to the double integral

$$\left\langle \hat{g}_{spiral}\left(H,D,r;t\right)\right\rangle_{z,\varphi}=\frac{1}{2}\int\limits_{1/\sqrt{4\alpha t}}^{\infty}\frac{du}{u}\int\limits_{D}^{H+D}\frac{dz'}{H}\Phi(Hu,Du,z'u)I_0\left(2u^2r\sqrt{\left(Pe/r_0/4/u^2\right)^2-\cos\left(z'k\right)Pe/2/u^2+r_0^2}\right)$$
$$\times\exp\left(-u^2\left[\left(r_0\cos z'k-Pe/4/u^2/r_0\right)^2+r_0^2 sin^2\left(z'k\right)+r^2\right]\right) \quad (15)$$

where $Pe=Ur_0/\alpha$ is the Péclet number defined for a heat source of a given radius r_0, $k=2\pi N/H$ is the wave number, and $\boldsymbol{N}$ –number of coil turns. The thermal response is scaled by the heat transfer rate per unit depth, i.e. q_z **,** which is related to the heat transfer rate per unit length of the spiral source q_l by $q_z=q_l\sqrt{1+r_0^2k^2}$.

Note that the thermal response function in the MFLS model is recovered for the zero radius of the spiral heat source at $\boldsymbol{r_0=0}$ in the proposed expression (16) and, therefore, the thermal response function in the MILS model for the limiting case of $H\to\infty$. Use of the new integral mean solutions for MFSS will result in a significant reducing CPU time in comparison with the numerical evaluating quadruple integrals [16].

3. Conclusions

Results have been presented of a study of the mean temperature for a GHE modeled as a moving finite line - source and moving finite spiral-source of a constant heat rate into a semi-infinite medium that presents a horizontal groundwater flow. The proposed lumped method consists of averaging the pile or borehole temperature, over the both azimuthal angle and vertical coordinate, rather than using its value at the middle point of the finite depth GHE with ground flow velocity in a radial direction. Thus, the mean temperature calculated by this method is smaller than the mid-point temperature in the both moving FSS and FLS source models. The MFLS predicts a lower value for the mean temperature than using MILS at any groundwater velocity in the heating mode.

We have provided new approximate expressions for the average thermal response function for the intermediate values of time in the vicinity of the moving finite line heat source. This may give insight into the duration of the thermal response test with groundwater and the time of steady-state attainment.

The solutions from the both MFLS and MFSS models have been verified: the result of the moving finite line-source model is recovered for the zero radius of the spiral heat source as well as the result of the FLS without

groundwater flow. The classical result of the traditional moving infinite line-source model is recovered for the intermediate values of time from the both MFLS and MFSS models. Taking into account axial component of the groundwater does not change the mean ground temperature in the MILS.

Results of the MFLS and MFSS models allow to evaluate the effect of groundwater flow on the temperature response in the ground and the pile. The analytical formulas for the mean temperature response as the function of the groundwater seepage velocity and geometry parameters might be helpful for analysis of thermal efficiency of a single energy pile and are suitable for the design of multiple borehole configurations.

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