Development and Analysis of the Mathematic Model of Optimal Balance Point for the Air Source Heat Pump Water Chiller/Heater

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Abstract

In this paper, firstly, based on a summary and analysis of heating character of air source heat pump, two conceptions of optimal energy balance point temperature and optimal economic balance point temperature are put forwards, then the calculation method of them given and mathematic model corresponding to them developed, according to the weather data and the prevailing price of equipment and energy source corresponding to the potential regions, with which two types of balance point temperature corresponding to potential regions can be obtained. At last, different balance point temperatures and their application setting Beijing as an example are given. So how to choose the water heater/chiller units of air source heat pump can be directed.

1 Introduction

As we all know, air source heat pump is influenced significiantly by weather when heating. Along with the outdoor air temperature falling, the quantity of heating is reduced step by step. Otherwise, the heat load of building gets more and more. When the heat load of building is equal to the heating quantity of air source heat pump, the outdoor air temperature is called as balance point temperature. When the outdoor air temperature is higher than balance point temperature, the heating quantity of air source heat pump is superfluous. The unit should be adjusted so that the heating quantity of it can be equal to heat load of building. Otherwise, the heating quantity is insufficient. In order to meet the requirement of heating, the assistant heat source is needed. If the balance point temperature we choose in design is too low, the capacity of the assistant heat source is too small to be needed. Yet the capacity of air source heat pump is too great. It runs at partial load for longer time, which causes its efficiency fall. On the other hand, If the balance point temperature is too high, the capacity of the assistant heat source too great, which is not favorable to energy saving. So economical efficiency, energy saving efficiency and how to confirm the capacity of the air source heat pump are influenced greatly by the appropriate balance point temperature. Therefore, in the paper, the development of mathematical model of the appropriate balance point temperatures in the potential regions for air source heat pumps are achieved judging by the Heating Seasonal Performance Factor (HSPF) according to the weather data and the prevailing price of equipment and energy source corresponding to regions, and then sensitivity analysis of them studied.

2 Development of Mathematical Model of Optimal Energy Balance Point Temperature

Study on the optimal energy balance point temperature of air source heat pump relies on the basis of the study on heating performance and character evaluation index of the unit of air source heat pump. At the same time, the optimal energy balance point temperature of air source heat pump is related to character of building envelop, heating load, weather condition etc (Zijie Wang. 1986). HSPF is recommended as character evaluation index of the unit of air source heat pump, which is accepted widely, by Department of Energy and Air Conditioning Refrigeration Institute of USA. Considering the character of the unit varying with weather, the BIN method is used to calculate HSPF. The optimal energy balance point temperature of air source heat pump in this paper is built on the basis of these research results (S.A.Tasson.1983), (RAYMOND.J. RETTBERG. 1983), (Nengzhao Jiang.1997).

The optimal energy balance point temperature is referred to as the balance point temperature of the maximum HSPF. HSPF is defined as follows (Bangyu Xu, Yajun Lu, and Zuiliang Ma. 1988):

$$HSPF = \frac{A}{B + C + D} \tag{1}$$

where A stands for the whole heating load of the building, B the whole energy consumption of the heat pump, C the whole energy consumption of the assistant heater, D the whole energy consumption in crankcase .

Besides the three parts above, the energy consumption for automatic control is so little as to be omitted. Now the calculated heating load of building and other energy consumption are substituted to equation (1), then the expression of HSPF is turned into as follows:

$$HSPF = \frac{\sum_{j=1}^{m} Q_1(T_j) \cdot n_j}{\sum_{j=1}^{m} K(T_j) \cdot W(T_j) \cdot n_j + \sum_{j=1}^{m} Q_a(T_j) \cdot n_j + \sum_{j=1}^{m} K(T_j) Q_e(T_j) \cdot n_j} \tag{2}$$

where:

$Q_1(T_j)$ = heating load of building at the temperature of T_j, (kW)

$Q_a(T_j)$ —energy consumption of assistant heater at the temperature of T_j,(kW)

$Q_e(T_j)$ —energy consumption for heating crankcase at the temperature of T_j, (kW)

$W(T_j)$ —energy consumption of heat pump at the temperature of T_j, (kW)

$K(T_j)$ —cut factor of heat pump at the temperature of T_j, (kW)

When the parameter of BIN, envelop character of building, indoor design parameter, outdoor design temperature of air conditioning, loss factor of frosting and defrosting and unit character of heat pump are determined in one region, the energy consumption of air source heat pump, assistant heater and crankcase heater is only related to the balance point temperature. So it can be concluded that HSPF is the function of the balance point temperature , which is given by:

$$HSPF = f(T_b) \tag{3}$$

From equation (3), T_b corresponding to the maximum HSPF has been calculated with exhaust algorithm. This temperature T_b is called as optimal energy balance point temperature.

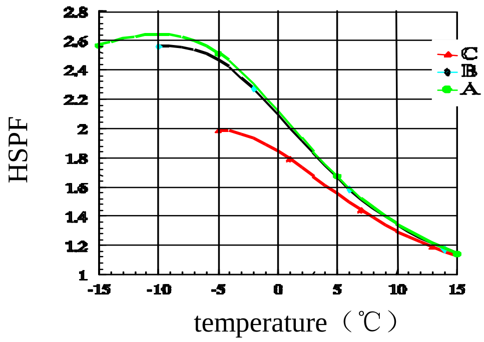
3 Sensitivity Analysis of Mathematical Model of Optimal Energy Balance Point Temperature

3.1 Effect on Optimal Energy Balance Point Temperature of Cut Temperature

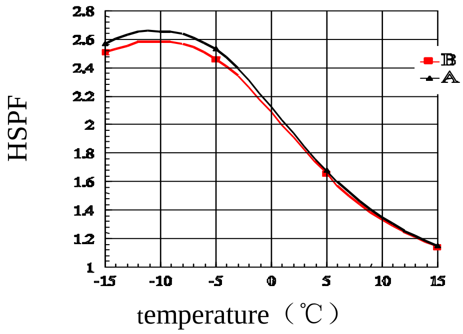
As shown in figure 1, the three curves illustrate the relation of balance point temperature and HSPF under the condition of different cut temperature, -15℃、-10℃、-5℃. Figure 1 shows: the cut temperature influences not only the optimal energy balance point temperature but also the HSPF corresponding to the optimal energy balance point temperature significiantly.

3.2 Effect on Optimal Energy Balance Point Temperature of Frosting and Defrosting Loss

As shown in figure 2, the curves of A and B illustrate the relation of balance point temperature and HSPF, where A stands for the condition considering the loss of frosting and defrosting and B without. Figure 2 shows: the optimal energy balance point temperature is independent to loss of frosting and defrosting fundamentally, but the HSPF corresponding to the optimal energy balance point temperature gets small.



Figue 1 Effcet on HSPF of cut temperature



Figue 2 Effcet on HSPF of loss of frosting and defrosting.

3.3 Effect on Optimal Energy Balance Point Temperature of Heat Transfer Coefficient of Building Envelop

In figure 3, the two curves illustrate the relation of balance point temperature and HSPF under the condition of different heat transfer coefficient of building envelop, K=1.97W/m^2.°C (the curve A), K=1.50W/m^2. °C (the curve B). Figure 3 shows: neither the optimal energy balance point temperature nor the HSPF corresponding to it is influenced by heat transfer coefficient of building envelop.

3.4 Effect on Optimal Energy Balance Point Temperature of Weather Character and Application Condition

In figure 4, the curve D illustrate the relation of balance point temperature and HSPF in Beijing under the condition that the unit runs at 8:00---18:00, and the curve B at 0:00---23:00, the curve C in Shanghai at 8:00---18:00, the curve A in Shanghai at 0:00---23:00. Figure 4 shows: the HSPF of Shanghai is more than that of Beijing, the HSPF under the condition running at 8:00---18:00 is more than that at 0:00---23:00. So it can be concluded that the optimal energy balance point temperature is influenced greatly by weather character and running time.

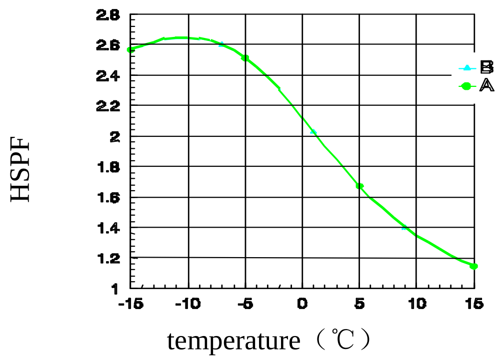


Figure3 Effcet on HSPF of building envelop

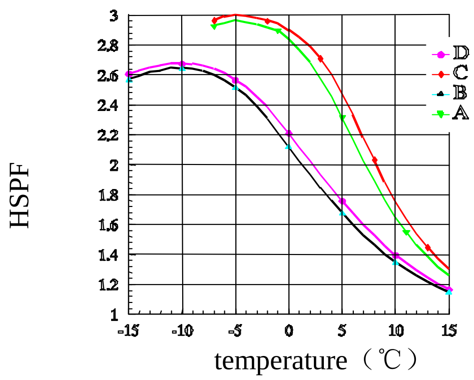


Figure4 Effcet on HSPF of weather feather

4 Development of Mathematical Model of Optimal Economic Balance Point Temperature

4.1 Conception of Optimal Economic Balance Point Temperature

As to one specified building, the capacity is relatively small and the initial investment and operating costs are relatively low for the selected unit of heat pump, but the assistant heat source is relatively large and the initial investment and operating costs of it are relatively high, when the balance point temperature selected is relatively high, and vice visa. From this, it is obvious that the amount of initial investment and operating costs is influenced singnificantly by the selected balance point of heat pump air conditioning system. One the other hand, now what a lot of owners concerned most is not energy saving, but money saving, i.e. relatively low initial investment and operating costs. So the optimal economical balance point is put forward, that is to say if the units of heat pump and assistant heat source is selected according to it, thus the lowest initial investment and operating costs of the whole air conditioning system can be obtained.

4.2 Initial Investment of Air Conditioning System of Air Source Heat Pump

Initial investment of air conditioning system of air source heat pump is generally divided into two parts which are initial investment for apparatus of the whole system and capacity affixed costs for power electricity and gas. They will be analyzed as follows:

(1) **Initial Investment of Apparatus** The apparatus of heat pump air conditioning system include the units of heat pump, assistant heat source, water pumps, and other control devices. It is assumed that the initial investment per unit area of the heat transportation devices and the control devices doesn't vary with the balance point. Therefore, the only initial investment of units of heat pump and assistant heat source such as oil boiler, gas boiler and electricity boiler is considered here, which is expressed as C_e.

(2) **Capacity Affixed Costs** It is divided into two parts which are capacity affixed costs of electricity power and gas. They are varying with region. Here it is expressed as C_z .

From above, the whole initial investment can be expressed as $C_t=C_z+C_e$.

4.3 Operating Costs of Air Conditioning System of Air Source Heat Pump

Operating costs of air conditioning system of air source heat pump depends on the type of heat source, whose prices are different from each other. Generally it includes four parts which are:

(1) **Operating Energy Consumption Costs** Operating energy consumption costs rely on the whole energy consumption, which was discussed in literature (Yiqiang Jiang. 1999), in the whole heating season. It is expressed as C_{yx}.

(2) **Basic Costs of Electricity** Basic costs of electricity are calculated on the basis of power per month (kW) or capacity of transformer (kVA). Generally, the charge is 6 to 10 Yuan ¥/(month .kW). Here it is expressed as C_{jb}.

(3) **Maintenance Costs of System** Now, maintenance costs of system day-to-day have not been specialized in literature. For the purpose of simpleness, it is calculated according to 1 percent of the whole initial costs, i.e. $C_{wx}=0.01 \cdot C_t$.

(4) **Management Costs of System** Management costs of system are looked on as the whole salaries gained by the operator and management worker, which is expressed as C_{gl}.

From above, the whole operating costs can be expressed as $C_r = C_{yx} + C_{jb} + C_{wx} + C_{gl}$.

4.4 Object Function of Optimal Economical Balance Point Temperature

From discussion above, the annual expense of air conditioning system of air source heat pump, can be expressed as follows (Guilan Wang.1990):

$$Aa = C_t \left[\frac{i(1+i^n)}{(1+i^n)-1} \right] + C_r \quad (4)$$

where:

i— annual interest rate;

n—duration of the apparatus;

Because that the initial investment per unit area of the heat transportation devices and the control devices doesn't vary with the balance point is assumed, the conception of annual expense per unit area is put forward and it can be expressed as follows:

$$Aas = Aa/F \quad (5)$$

where:

Aas——annual expense per unit area，Yuan/(m^2.Year);

Aa——annual expense of air conditioning system of air source heat pump，Yuan/year;

F——building area，m^2.

From analysis above, it is obvious that the object function varies with the amount of variables. But, in fact, once the annual interest rate, duration of the apparatus, and the prices of apparatus and energy source are confirmed, it becomes the function only varying with balance point. Obviously, the balance point temperature corresponding to the minimum annual expense per unit area is the optimal economic balance temperature. So, the program about the optimal economic balance temperature can be developed and the optimal economic balance temperature can be obtained.

5 Illustration and Analysis

The mathematical models for the optimal energy balance point temperature and the optimal economic balance point temperature have been given, and consequently the two types of balance point temperature will be calculated and their application illustrated, setting Beijing as an example.

Table 1 all kinds of correlative parameters in Beijing

heat transfer coefficient of building envelop w/(m^2. ℃)	Indoor design parameters ℃	Outdoor calculation temperature in winter ℃	Electricity price Yuan/（kW .h）	Electricity Capacity affixed costs Yuan/kW	Gas price Yuan/ m^3	Gas Capacity affixed costs Yuan/(m^3/h)	Frosting and defrosting loss coefficient
1.5	18	-12	0.462	5800	1.2	600	0.965

The above parameters are substituted into mathematical models, the optimal balance point temperature can be calculated, i.e. the optimal balance point temperature relying on electricity boiler as assistant heat source is -11 ℃, the optimal economic balance point temperature 4 ℃, and the optimal economic balance point temperature relying on gas boiler assistant as heat source 11 ℃.

Therefore, the water heater/chiller units of air source heat pump and assistant heat source can be confirmed very conveniently, according to your own attention. For example, to one building whose heat load is 1000kW around, if the designer pay much more attention to maximum HSPF (Heating Season Performance Factor), the capacity of the unit is confirmed depending on the optimal energy balance point temperature, which is 966kW under the temperature of -11 ℃. On the other hand, if the designer pay much more attention to money-saving, the capacity of the unit will be confirmed depending on the optimal economic balance point temperature, which is 733kW under the temperature of 4 ℃. All those considered above depend on electricity boiler as assistant heat source and if the assistant heat source is changed into gas boiler, the capacity of the heat pump unit will be confirmed in the same way for maximum economy benefits. In a word, the blindness to choose heat pump units is prevented according to the optimal balance point temperature. To our attention, the optimal energy balance point temperature is influenced by local weather characteristics and the optimal economic balance point temperature by energy price, capacity affixed costs and types of assistant heat source etc.. The optimal energy balance point temperature and the optimal economic balance point temperature in other regions in China can be seen in literature (Yiqiang Jiang. 1999).

6 Discussion

In the course of mathematic model development for the optimal energy balance point temperature and the optimal economic balance point temperature, the special weather (e.g. windy, rainy, snowy) is not considered, and the model will be affected to some extent.

7 Conclusion

(1) From the point of view of HSPF, the optimal energy balance point temperature is put forward, and the mathematical model corresponding to it developed. The maximum HSPF can be got if the water heater/chiller units of air source heat pump are selected according to the optimal energy balance point temperature.

(2) From the point of view of initial investment and operating costs, the optimal economic balance point temperature is put forward, and the mathematical model corresponding to it developed. The maximum economy benefits can be obtained if the water heater/chiller units of air source heat pump are selected according to the optimal economic balance point temperature.

(3) The application of the energy economic balance point temperature and the optimal economic balance point temperature is illustrated, setting Beijing as an example. The water chiller/heater units of air source heat pump and assistant heat source can be confirmed conveniently according to our own attention and the blindness to select them is prevented.

8 Acknowledgments

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