The beauty of reason and insight: a story about 30 years old borefield equations

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Equations rewrite the world around us in the language of mathematics. They are implemented in a lot of tools used by numerous scientists, engineers and practitioners. They are everywhere, and yet we sometimes forget to take a step back and look at these equations at face value. What can they teach us? In this article, we try to convince you of the beauty of borefield equations and how these equations on their own can help us to reason about the question of whether or not it is useful to divide a thermal load among two borefields.

Equations are useful. Any scientist or engineer will confirm this statement. Equations rewrite the world around us in the language of mathematics, so we can use them to solve problems. Hence, they provide a tool that we can use to learn about the world. But what about the equations themselves? Are the direct insights they offer often overlooked?

This article tells you the story of borefields, but now, the equations themselves are the protagonist. After more than 30 years of research into their mathematical complexity and finding new ways to make them easier to compute, we pause and look at them directly. Can we still learn something from these 30 years old equations? Can we answer the question of how a thermal load should be divided among two borefields without doing detailed simulations?

History of the protagonist: g-functions

Borefields can be used to heat and cool our (residential) buildings in a sustainable way, by exploiting the heat/cold in the ground. However, designing such borefields is quite challenging, since it is a three-dimensional transient heat transfer problem.

In 1987, Eskilson came up with the idea of g-functions to solve this complex problem [1]. g-functions show the temporal temperature evolution of the borehole wall for a constant heat extraction in a non-dimensional form. By applying temporal superposition of multiple constant loads to obtain the real borefield load, an accurate calculation of the borehole wall temperature can be found. Figure 1 (left) represents the thermal step response of the borehole wall temperature when a constant heat injection is applied. This corresponds to the non-dimensional g-function (in semi-log scale) on the right.

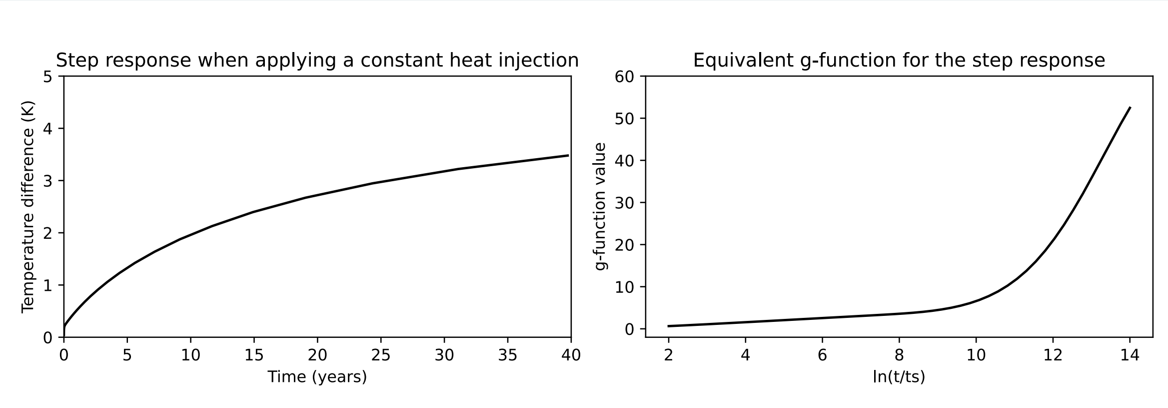

Figure 1. Thermal step response (left) and the corresponding g-function (right) with t_s = 150/9ks (k_s is the ground t hermal conductivity in W/mK)

Thanks to these g-functions, it is possible to take into account both the internal thermal interactions between boreholes in a borefield and the external interactions with the surrounding ground. Previously, numerous different temperature penalties were used to obtain these interactions [2]. Although there is still a lot of research going on how to calculate these g-functions as fast as possible (see [3] for a novel development and [4] for a recent overview), our story starts from the original idea of Eskilson.

Eureka! The most wonderful thoughts of my life

The insights presented in this article originally arose from the question: If there are two borefields connected to each other, how should we divide the load of the consumers among both fields while minimizing the total borefield length (i.e. the sum of the length of all boreholes) [5]? As practitioners typically define the borefield cost based on the total borefield length, a minimal borefield length corresponds to a minimal investment cost. Without using detailed simulations, reasoning based on the g-functions helped us answer this question.

Let's start with Figure 2 which represents the g-functions of a 10x12 borefield for multiple borehole depths. From these g-functions, the thermal resistance between the borefield and the surrounding ground (R_a) for a design period of 20 years can be derived. This thermal resistance is linearly proportional to the difference in g-function values between two points on the horizontal axis, 20 years apart (as indicated in Figure 2). The reasoning in this article is based on the long-term effects only and hence takes only the yearly imbalance into account. Although for design purposes peak loads are also important, they can be neglected (without loss of generality) for this exercise [5]. Note that the borehole depth is constrained by practical design limitations, i.e. a minimum depth of 50 m and a maximal depth of 350 m are assumed. A g-function is a unique characteristic of a borefield and depends on the configuration and the depth of the boreholes. Given a defined configuration of the boreholes, the thermal resistance R_a of that borefield can be plotted as a function of the depth of the boreholes. This results in Figure 3 represent the resistance R_a for different square borefields. This figure shows that for an equal borehole depth, more boreholes lead to a larger thermal resistance R_a meaning that it is more difficult for the borefield to exchange heat with the ground. This is because the boreholes in the borefield affect each other. The more boreholes, the more mutual thermal interactions are present.

Subsequently, it is possible to derive the relation between the depth of the boreholes and the imbalance at which a certain borefield reaches its design limits. This

$$L = \frac{q_y.R_y + q_m.R_m + q_h.R_h + q_h.R_b^*}{T_m - T_g} \quad (Eq.1)$$

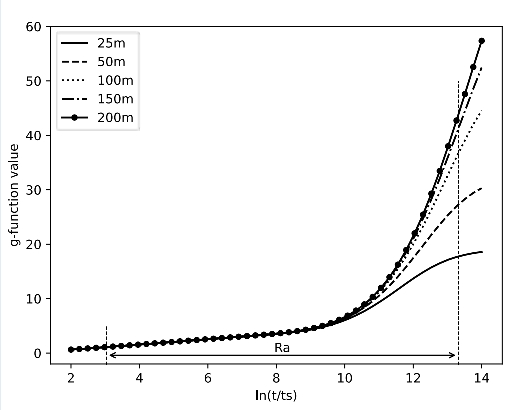

Figure 2. g-functions of a 10x12 borefield for multiple borehole depths (t_s=150/9k_s, where k_s is the ground thermal conductivity in W/mK)

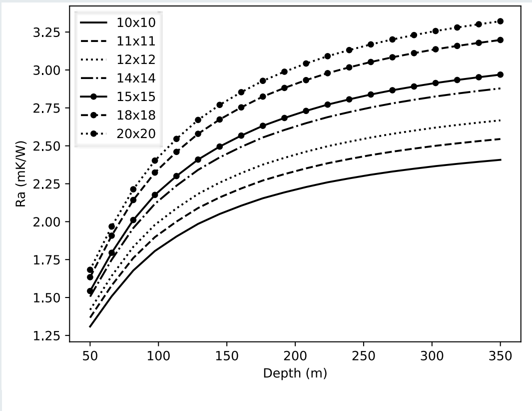

Figure 3. Relation between R_a and borehole depth for multiple borefields

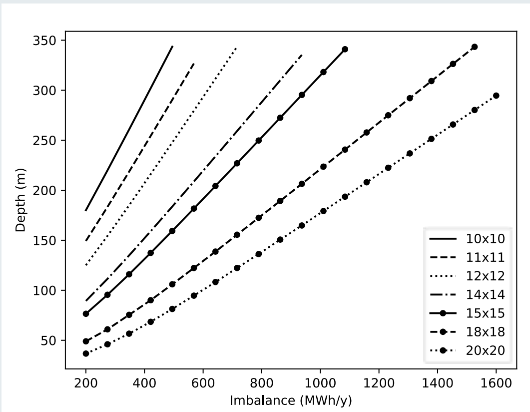

Figure 4. Relation between borehole depth and imbalance for multiple borefields

relationship is derived using the L2-sizing method (see Eq. 1) and is represented in Figure 4. In Equation 1, the q_i's are heat pulses, the R_i's are thermal resistances (calculated with g-functions) and the T_i's are temperatures (for more information we refer to [2, 5, 6]).

The yearly term $q_y.R_y$ is significantly larger than the other terms, which justifies the simplification of neglecting the other terms. Figure 4, shows that for an equal borehole depth, the more boreholes, the more imbalance a borefield can dissipate in the ground. Note that the number of boreholes multiplied by the borehole depth gives the total borefield length, which scales with the imbalance.

From Figure 3 and Figure 4, it is possible to eliminate the depth variable and to compose a plot that represents the thermal resistance R_a as a function of the imbalance. This results in Figure 5. Eureka! This graph gives useful insights into the design of borefields. Firstly, we see that the curves intersect each other, even though this is not the case in Figure 3 and Figure 4. This is because of the ratio between the total borefield length (a larger borefield length can dissipate a larger imbalance) and the surface-area-to-volume ratio of the borefield (a larger surface-area-to-volume ratio means less mutual thermal interactions). Secondly, we can answer the question posed in the introduction of this article: If there are two borefields connected to each other, how should we divide the load of the users among both fields while minimizing the total borefield length? This question can be discussed for two cases; the case with two identical borefields and the case with two different borefields.

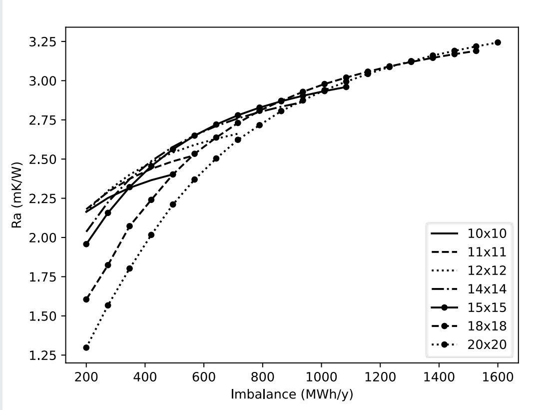

Figure 5. Relation between R_a and imbalance for multiple borefields

Two identical borefield configurations

Consider a situation where a thermal load is coupled to two identical borefield configurations. The question now is whether it is better to divide the load equally among both fields or not. The answer can be found in Figure 6, which is a general representation of the thermal resistance R_a as a function of the imbalance. If the load is divided equally, both fields experience an imbalance *I* and consequently have a thermal resistance R_a. Imagine now that part *Q* of the imbalance *I* is shifted from the first to the second field. Then, the first field experiences an imbalance *I-Q* and the second field an imbalance *I+Q*. The thermal resistances are $R_{a,2}$ and $R_{a,2'}$ respectively. The average thermal resistance $R_{a'}$ is now smaller than the thermal resistance R_a for a borefield experiencing an imbalance *I*, so one would expect a smaller borefield length, right? However, one should look at the total borefield length (as we are minimizing this value). From Equation 1, we know that the thermal resistance is weighted with the imbalance. Hence, the total borefield length in the unequal case $L_{22'}$ is always larger than the borefield length in the equal case L_{11}, because

$$L_{22'} \sim L_{11} + R.Q - R'_a.I \quad (Eq. 2)$$

And

$$R.Q - R'_a.I \geq 0 \quad (Eq. 3)$$

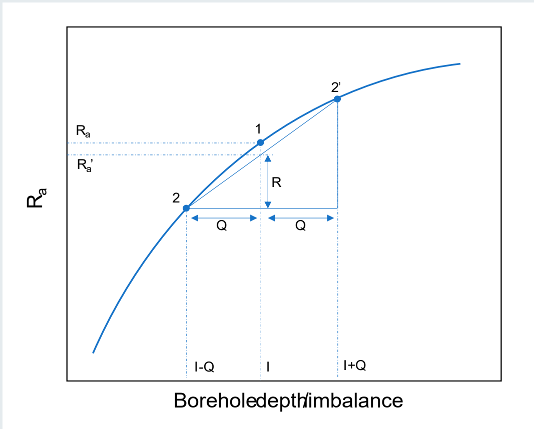

Figure 6. General representation of the relation between R_a and the imbalance

This means that an equal division of the imbalance is always better than an unequal division [5].

Two different borefield configurations

It could also happen that two different borefield configurations are present. So the same question as before arises: is it better to divide the imbalance equally among two different borefields or not? For this question, again, plotting the thermal resistance R_a as a function of the imbalance can give an answer. Consider Figure 7, which represents the relation between R_a and the imbalance for two borefields (borefield 1: 15x15, borefield 2: 7x15). Both curves intersect. If the total imbalance (load) of the whole system is larger than the imbalance at the intersection, then it is better to put as much as possible of the imbalance on the smaller borefield (borefield 2) (as long as borefield 1 reaches its design criterion in the last year [5,6]). If the imbalance is smaller than the one at the intersection, it is better to impose all imbalance on the

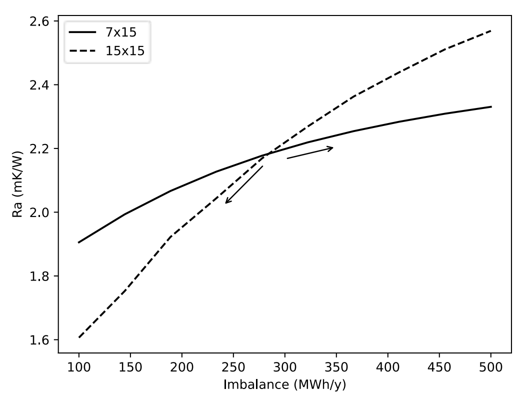

Figure 7. Thermal resistance as a function of the imbalance for two different borefields

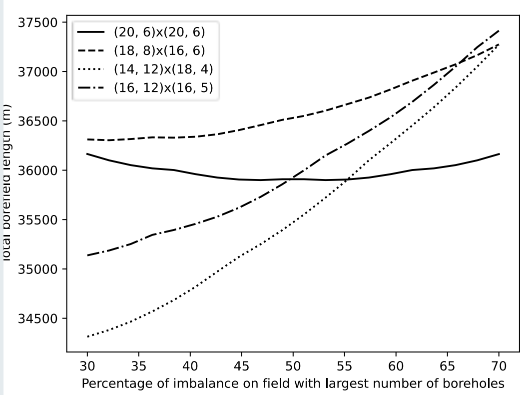

Figure 8. Effect of the imbalance distribution on the total borefield length

larger field (borefield 1). Note that the curves in Figure 7 will change (and so the intersection too) if the number of boreholes or the configuration changes. However, the same method applies.

The previous insights could be merged in Figure 8 which represents the total borefield length for several combinations of borefields, depending on the division of the imbalance (800 MWh/y) among both fields. Firstly, one can see the results in the case of two identical borefields. An equal division corresponds to 50% on the x-axis. If more imbalance is shifted towards one field, the total borefield length increases. However, the differences are rather small. The other lines represent combinations of different borefields. The cases in Figure 8 show that it is better to put as much imbalance as possible on the smallest borefield (until its design limitations are met or the larger borefield reaches its temperature limitation in the first year of operation [5, 6, 8]). This is because the intersections of all represented cases (corresponding to Figure 7) are smaller than the total imbalance of the two borefields together.

Conclusions

The main purpose of this article is to show that reasoning about equations often gives useful insights in and understanding of a problem, without using detailed simulation tools. In this specific case, we answered the question of how a thermal load should be divided among two borefields. In the case of two identical borefields, it is always better to divide the load equally. In the case of two different borefields, it depends on the total imbalance because the thermal resistance – imbalance curves of the two borefields show an intersection. All figures in this article were created with the open-source borefield sizing tool GHEtool [7].

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