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Methodology to evaluate the uncertainty on the design length calculation of vertical ground heat exchangers

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Abstract

In the first part of this study, various calculation methods to determine the design length of vertical ground heat exchangers are reviewed and compared to each other using five test cases. This comparison indicates that one can expect a ±10% accuracy on calculation methods. Then, two calculation methods (L2 and L4) are used to quantify the uncertainty in the design length L. The L2 method is based on the ASHRAE three-pulse equation while the L4 method uses hourly simulations. The uncertainty is quantified using two methods. The first one evaluates a local uncertainty using an uncertainty propagation technique. The second gives a global uncertainty and is based on Monte-Carlo-type simulation where significant parameters are varied according to a statistical distribution.

In the evaluation of the local uncertainty, the impact of individual uncertainties on 7 parameters (ground thermal conductivity and diffusivity, three ground loads (peak, monthly, annual) undisturbed ground temperature and borehole thermal resistance) are examined for five test cases using a L2 method. When a ±1°C uncertainty is assigned to the undisturbed ground temperature and a ±10% uncertainty on the six other parameters, the resulting local uncertainty varies from ±11.5% to ±14.2% for the five test cases.

A total of 1024 simulations using a Sobol sequence are performed on L2 and L4 methods to obtain the global uncertainty on L for all test cases. Results obtained with the L2 methods range from ±11.4% to ±14.4% for all five test cases and are thus very similar to the results obtained with local uncertainty calculations. The potential impact of uncertainties on the outlet fluid temperature from the borehole field is also examined. It is shown that the uncertainty on the outlet fluid temperature at peak conditions is of the order of ±2°C.

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Keywords: Ground heat exchangers; uncertainity; borehole field; sizing

1. Introduction

The uncertainty associated with the design length L of vertical ground heat exchangers (VGHX) is rarely calculated even though it could provide some valuable risk-related information for designers. The calculated design length of a VGHX is uncertain mainly because of two factors. First, the calculation methods and their associated software tools might be inaccurate. Secondly, design parameters used to calculate L might be uncertain. This article examines these two factors to determine what accuracy is expected from software tools and what is the global uncertainty when design parameters are uncertain.

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2. Literature review

The impact of errors on ground thermal conductivity and diffusivity on the design length was quantified by Allan and Kavanaugh [1,2]. In one example, it was shown that the required loop length increased by 5.8% with a 10% decrease in ground thermal conductivity and diffusivity. If the design length is installed and the ground thermal conductivity and diffusivity are 10% lower than expected then the loop will operate at slightly higher temperature (+1.2°C) in cooling while the heat pump capacity and COP would be lowered by 1% and 2%, respectively.

Bernier [3] presents an uncertainty analysis on the VGHX design length for a particular case using the classic ASHRAE sizing equation (which uses the concept of temperature penalty). Sensitivity coefficients are calculated for a particular case, and it is shown that the ground thermal conductivity and the peak hourly ground load are the two most important parameters in the determination of the design length. Then, individual uncertainties of each parameter are combined using a classic uncertainty propagation technique to obtain a local uncertainty.

Peere [4] shows the problems associated with using rules of thumb for borehole field sizing. Using a VGHX sizing tool the author varied the important design parameters in what appeared to be a Monte-Carlo-type analysis. As expected, results show that rules of thumb are inadequate. Furthermore, the spread in the total borehole length obtained by varying a certain number of parameters is important. However, no details are given on how the parameters were varied.

Pertzborn et al. [5] studied the impact of yearly weather variation on ground-source heat pump design. They found that results obtained using real weather files to calculate building loads are significantly different from those obtained using a typical meteorological year for every year of the design period.

Dusseault and Pasquier [6] present a design-optimization method to minimize the net present value-at-risk of ground-source heat pump systems. Their approach necessitated the evaluation of the probability distribution of the net present value due to uncertain parameters, namely the construction costs, the electricity costs, the ground thermal conductivity and the building loads. Random samples were generated for uncertain parameters, assuming an appropriate statistical distribution of each of the parameters. They found that accounting for the uncertainties of parameters during design improves the profitability of the system, on average.

The methodologies to identify the influence of design variations on building energy performance are somewhat similar to the ones proposed here. These methodologies have been reviewed by Bucking et al. [7] and Hopfe and Hensen [8]. De Wit and Augenbroe [9] showed the effect of variations in heat transfer and climate variables on thermal comfort and energy consumption to facilitate rational design decisions under uncertainty. Lomas and Eppel [10] recommended differential sensitivity techniques for sensitivity predictions in building simulation programs. Hopfe [11] made clear distinctions between local and global methods, both of which will be used in this study.

3. Uncertainties from sizing calculation methods

Methods to evaluate the required design length of VGHX are well established [12,13]. In their review of existing methods, Ahmadfard and Bernier [14] classified design methods into five categories (L0 to L4). Higher level methods (L2, L3, and L4) were analysed and corresponding software tools that use these methods were identified. L2 methods are based on various versions of the three-pulse ASHRAE equation [15], L3 methods involve monthly average and peak pulses [16], and L4 methods use annual hourly simulations.

A series of test cases were developed by Ahmadfard and Bernier [14] to perform an inter model comparison of these various calculation methods. These test cases cover a large spectrum of ground load conditions and borehole field configurations. The main parameters used in these tests are shown in Table 1 where B is the distance between boreholes, k is the ground thermal conductivity, α is the ground thermal diffusivity, T_g is the undisturbed ground temperature, Q_h is the peak ground load, Q_m is the average monthly ground load when Q_h occurs, Q_a is the annual ground load imbalance in the ground, R_b is the equivalent borehole thermal resistance, and T_{out} is the maximum outlet temperature from the borehole field (i.e. the inlet temperature to the heat pump). The peak load duration is 6 hours for L2 and L3 methods. All tests are performed with a 10-year design period except for Test 4 which uses 20 years.

Table 1. Main parameters used in the inter model comparison study of Ahmadfard and Bernier [14]

	Number of boreholes	B (m)	k (W/m·K)	α (m²/day)	T_g (°C)	Q_h (W)	Q_m (W)	Q_a (W)	R_b (m·K/W)	T_{out} (°C)
Test 1A	1	-	1.8	0.075	17.5	4428	648	1	0.13	35
Test 1B	1	-	1.8	0.075	17.5	5586	818	120	0.13	35
Test 2	12×10	6	2.25	0.068	12.41	-395127	-100003	-1763	0.113	4.4
Test 3	7×7	5	2.25	0.075	10	-238670	-105374	7712	0.1	0
Test 4	5×5	8	1.9	0.08	15	139731	46983	19968	0.2	38

A total of 14 calculation methods/software tools were compared for each test (seven L2 methods, five L3 methods, and two L4 methods). Three methods included borehole thermal capacity in the calculation. For Test 4, one software tool was discarded from the analysis as it clearly had a calculation error. The mean borehole field length obtained with these 14 methods are given in Table 2 as well as the standard deviation of the mean. Based on the standard deviation, there is relatively good agreement among the software tools except for Test 3. Test 3 was purposely designed so that the design length occurs during the first year of operation. Some software tools (L2 methods) cannot handle this situation accurately which explains the large value of the standard deviation for Test 3.

Table 2. Results of an inter model comparison for 14 software tools

Test	Mean value of H (m)	Mean value of L (m)	σ, Standard deviation on H (m) (% of H)
Test 1A	60.0	60.0	2.5 (4.2%)
Test 1B	76.3	76.3	3.0 (4.4%)
Test 2	89.1	10688	4.1 (4.6%)
Test 3	99.7	4887	12.0 (12.0%)
Test 4	120.9	3023	6.4 (5.3%)

In summary, if Test 3 is excluded, the standard deviation (σ) of the other tests is approximately equal to 5%. Using 2σ as an indicator of the accuracy, one can expect a ±10% accuracy on the software tools.

4. Methods/approach

The impact of the uncertainty of several design parameters will now be examined with the goal of estimating an overall uncertainty which would encompass the contribution of each parameter.

The test cases established by Ahmadfard and Bernier [14] and presented in the previous section are used for this analysis. Table 3 lists the parameters that will be examined and their assumed uncertainties which represent two standard deviations. The methods presented below are not restricted to these assumed uncertainties and other uncertainties could be used. The uncertainty of the depth of the borehole head is not considered in this study as well as the duration of the peak load, the starting date of the system, and the borehole verticality.

Table 3. Parameters examined in this study and their associated uncertainties

Parameter	Symbol (x_i)	Units	Uncertainties ($2\sigma_{x_i}$)
Ground thermal conductivity	k	W/m·K	$\pm$ 10 %
Ground thermal diffusivity	α	m²/sec	$\pm$ 10 %
Undisturbed ground temperature	T_g	°C	$\pm$ 1 ℃
Peak hourly ground load	Q_h	W	$\pm$ 10 %
Monthly ground load	Q_m	W	$\pm$ 10 %
Annual ground load	Q_a	W	$\pm$ 10 %
Borehole position	-	m	$\pm$ 0.5 m
Borehole thermal resistance	R_b	m·K/W	$\pm$ 10 %

4.1. Sizing calculations

The overall uncertainty on the total required borehole length is evaluated using two approaches. The first one revisits the uncertainty propagation technique described by Bernier [3] to obtain local uncertainties. The second approach uses a Monte-Carlo type of simulation to obtain the global uncertainty. These methods are described in sections 4.2 and 4.3.

Two sizing approaches based on L2 and L4 methods were developed and used in the evaluation of the local and global uncertainties. These approaches will now be described.

4.1.1. ASHRAE sizing equation (L2 method)

The ASHRAE three-pulse sizing equation for VGHX based on g-function [13, 17] is given by:

$$L = (H \times nb) = \frac{Q_a R_{a,g} + Q_m R_{m,g} + Q_h \left[R_{h,g} + R_b\right]}{\frac{(T_{in}+T_{out})}{2} - T_g} \tag{1}$$

where L is the total required borehole field length, H is the height of individual boreholes, nb is the number of boreholes, and T_{in} and T_{out} are the inlet and outlet temperatures at design conditions. The equivalent thermal resistances based on g-function values are given by:

$$R_{a,g} = \frac{g_{(t_3-0)} - g_{(t_3-t_1)}}{2\pi k} \quad , \quad R_{m,g} = \frac{g_{(t_3-t_1)} - g_{(t_3-t_2)}}{2\pi k} \quad , \quad R_{h,g} = \frac{g_{(t_3-t_2)}}{2\pi k} \tag{2}$$

g-Functions are evaluated for three time intervals: $(t_3 - 0), (t_3 - t_1)$, and $(t_3 - t_2)$ with values of 3680.25 (except for Test 4 with 7330.25), 30.25, and 0.25 days. Thus, the peak load duration (t_3-t_2) is assumed to be 6 hours as per ASHRAE's recommendation [15]. This is open to debate and other researchers have suggested evaluating the peak load and its duration using a hybrid time step [18]. The peak load duration influences $R_{m,g}$ and $R_{h,g}$ and consequently the required borehole field length. The work presented here could eventually be extended to include an uncertainty on the peak load duration.

When designing a borehole field, Eq. (1) is solved in heating and cooling, with the longest length representing the design length. An iterative process is required as the equivalent borehole thermal resistances and the g-function values depend on H, which is unknown *a priori*. Eq. (1) can be rearranged to obtain T_{out} at the design condition for a given value of L.

$$T_{out} = T_g - \frac{Q_h}{2\dot{m}c_p} + \frac{Q_a R_{a,g} + Q_m R_{m,g} + Q_h \left[R_{h,g} + R_b\right]}{L} \tag{3}$$

where $\dot{m}$ is the total mass flow rate and c_p is the fluid specific heat capacity.

4.1.2. Sizing based on hourly simulations (L4 method)

The sizing of a borehole field using hourly simulations consists in finding the minimum length that maintains the outlet fluid temperature above the minimum design temperature and below the maximum design temperature. At any time t_n, the outlet fluid temperature is obtained from the temporal superposition of the g-function. The total required borehole length is thus defined by an optimization problem:

$$\begin{aligned} \boldsymbol{min} \quad & f(H) = H \times nb \\ \boldsymbol{s.t.} \quad & T_{out,n} = T_g + q_n R_b - \frac{q_n L}{2\dot{m}c_p} + \sum_{i=1}^{n} \frac{(q_i - q_{i-1})}{2\pi k}\, g(t_n - t_{i-1}) \\ & \min_n T_{out,n} \geq T_{out,min} \\ & \max_n T_{out,n} \leq T_{out,max} \end{aligned} \tag{4}$$

where q_i is the ground load per unit borehole length during the interval $t_{i-1} < t \leq t_i$ (positive for heat injection), $T_{out,n}$ is the outlet fluid temperature at time t_n, and $T_{out,min}$ and $T_{out,max}$ are the minimum and maximum design temperatures. It should be noted that the g-function is also a function of the borehole length H.

A frequency-domain convolution using forward and inverse Fast Fourier Transforms is utilized to evaluate $T_{out,n}$ at all times n in a single operation, as proposed by Marcotte and Pasquier [19]. Since one of the temperature limits is always active at the minimum length, the optimization problem of Eq. (4) can be conveniently solved using a root finding algorithm. Here, the modified Powell method implemented in the `scipy.optimize` module of `scipy` [20] is employed to find the root of the function:

$$f'(H) = min\left(\min_n T_{out,n} - T_{out,min},\ \ T_{out,max} - \max_n T_{out,n}\right) \tag{5}$$

As shown in Eqs. (1) and (4), the accurate determination of ground loads is crucial to obtaining a good estimate of the required borehole length. In this work, it is assumed that the same annual ground loads are repeated from year to year. Situations where ground loads are uncertain and vary over the years due to changing weather conditions, occupancy, etc. are outside the scope of the present analysis. If one deems that the ground loads over time are uncertain, then the assumed uncertainties should be increased accordingly. If multi-year estimates of ground loads are available, they can be used directly for hourly simulations in Eq. (4), or ground loads in Eqs. (1)-(3) should be adjusted to reflect the ground load history at peak conditions if the ASHRAE three-pulse equation is used.

4.1.3. Verification of the calculation methods

The calculation methods represented by Eqs. (1)-(5) were implemented in Python scripts using `pygfunction` [21] as the calculation engine when evaluating g-functions. The results obtained with these scripts are compared against the results presented in Table 2 for the five test cases. The results of this comparison are given in Table 4. It is shown that the L2 and L4 methods developed for this study give results that are comparable to the results of the inter model comparison and that they can be used with confidence.

Table 4. Differences among borehole lengths *H* for results of Table 2 (mean of 14 tools) and L2 and L4 methods developed for the present analysis

Borehole heights H	Table 2 results (m)	L2 method (3-pulse) (m)	L4 method (hourly) (m)
Test 1A	60.2	59.4	56.9
Test 1B	76.3	76.1	72.8
Test 2	89.1	86.2	91.3
Test 3	99.7	89.6	111.3
Test 4	120.9	121.3	120.3

4.2. Local uncertainty using uncertainty propagation

This technique is based on the well-known uncertainty propagation technique used to quantify measurement uncertainty [22]. It was used by Bernier [3] and is presented briefly below. Assuming that the individual parameters are uncorrelated and random, the local uncertainty on L, U_L, is given by:

$$U_L = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial L}{\partial x_i}U_{x_i}\right)^2} \tag{6}$$

where U_{x_i} is the uncertainty on parameter x_i and n is the number of parameters. The local sensitivity coefficients, $\partial L/\partial x_i$, are computed here using the following approximation of the partial derivative using a small perturbation, Δx_i, of each variable x_i:

$$\frac{\partial L}{\partial x_i} \approx \lim_{\Delta x_i \to 0} \frac{L_{x_i+\Delta x_i} - L_{x_i}}{\Delta x_i} \tag{7}$$

Eq. (7) can be used to determine which variable x_i has the strongest impact at point L. It is thus a local impact. It does not show how sensitivities behave globally under wider variations. A

4.3. Global uncertainty using Monte-Carlo simulations

This technique uses random sampling of parameters to evaluate the impact of design parameters on the total required borehole length. Samples of sets of parameters from Table 3 are generated, assuming normal distributions, and L2 and L4 methods are used to evaluate the total required borehole length for each sampled set of parameters. The uncertainty on the total required borehole length is then evaluated from a 95% confidence interval, approximately equal to twice the standard deviation σ_L of the evaluated total borehole lengths. Details of the random sampling are included in the results sections 5.1 and 5.3.

5. Results and discussion

5.1. Impact of borehole positions (L2 method)

The impact of borehole positions on the required borehole length is first assessed. The analysis is only applied to Tests 2, 3 and 4, since Tests 1A and 1B include only a single borehole; therefore, its position does not affect the required borehole length. From Table 3, individual positions of the boreholes on square grids are varied by an offset with a standard deviation of $\sigma_{position}$ = 0.25 m (half of the uncertainty) in a random direction assuming a normal distribution. A Sobol sequence, as implemented in the `scipy.stats.qmc` module of `scipy` [20], is utilized to generate 1024 samples of nb offsets for the boreholes. Each of the generated offsets are associated with a random direction on the horizontal plane (between 0 and 2π) assuming a uniform distribution. The required borehole length evaluated using the L2 method of Eqs. (1)-(2), is evaluated for each of the generated samples.

Figure 1 shows the distributions of the required borehole length H for Tests 2, 3 and 4. The horizontal blue lines correspond to the maximum, mean and minimum values of the required borehole length. Each of the red dots on the right-hand side of the distributions corresponds to the required length from one of the 1024 samples. The impact of borehole positions is small in all three tests, with standard deviations of σ_H = 0.01 m for Tests 2 and 3, and σ_H = 0.09 m for Test 4. The small impact is explained by the fact that an offset in the position of a borehole brings it closer to some of its neighbours but farther from others, and the overall impact on the thermal interactions in the borehole field thus tends to cancel out. Comparing tests 2 (120 boreholes), 3 (49 boreholes) and 4 (25 boreholes) shows that these small impacts average down to a negligible impact on the required borehole length when the number of boreholes increases.

For the remainder of the analysis, the uncertainty of the borehole positions will be neglected, which brings down the number of parameters to a more manageable scale. Only the 7 remaining thermal parameters of Table 3 will be considered, down from $7 + nb$ parameters.

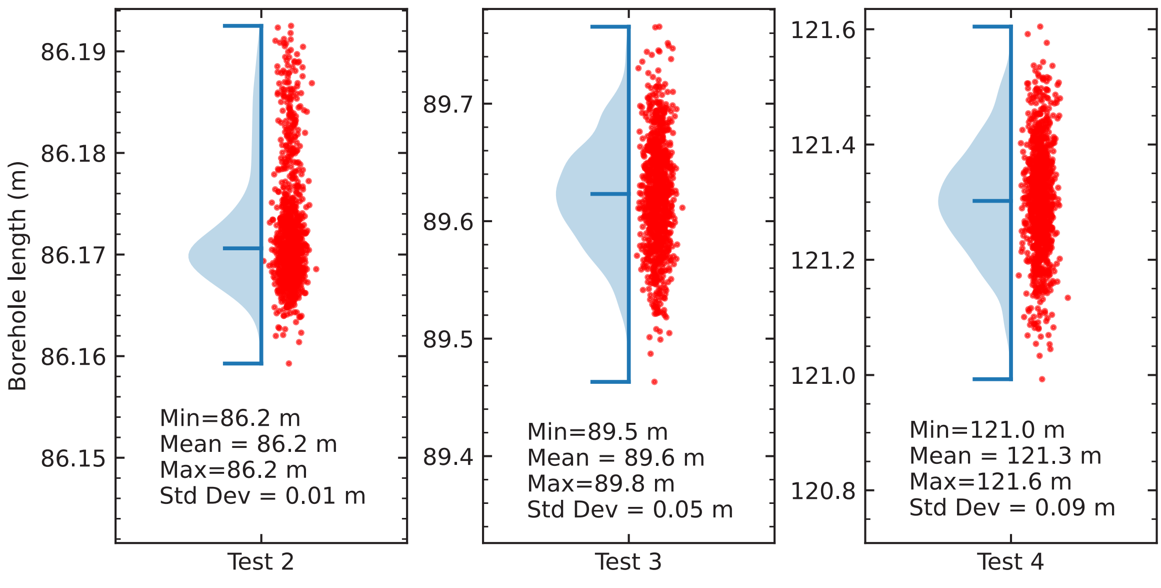

Figure 1. Distributions of the required borehole length H for Tests 2, 3 and 4 due to the uncertainty on the borehole positions using a L2 method

5.2. Local uncertainty propagation (L2 method)

The local uncertainty propagation technique presented in section 4.2 is applied to the L2 method described by Eqs. (1)-(2), using the borehole lengths reported in Table 4 for the L2 method. To evaluate the sensitivity coefficients using Eq. (7), a relative variation $\Delta x_i/x_i$ of 1 % is applied to all thermal parameters of Table 3 except for the undisturbed ground temperature T_g where an absolute variation ΔT_g of 0.01°C is used. Based on the results of section 5.1, the impact of the borehole positions is neglected.

Table 5 presents the contribution of each parameter to the overall, locally evaluated, uncertainty on the total required borehole length. The last column presents the local uncertainty obtained using the uncertainty propagation method of section 4.2. For each test case, the parameter with the largest contribution is highlighted in green and the parameter with the smallest contribution is highlighted in yellow. The local uncertainty varies from ±11.5% in Test 4 to ±14.2% in Test 2. The relative contributions vary from 0% for Q_a in Test 1A to 10.3% for T_g in Test 2. The contributions of each parameter vary across tests, and there is no clear trend as to which of the parameters have the single most and least significant impact on the overall uncertainty. However, the ground thermal conductivity k, the undisturbed ground temperature T_g, the peak ground load Q_h and the borehole thermal resistance R_b all have contributions that surpass 5% in at least one of the tests.

Table 5. Contributions to the overall uncertainty assuming a ±10% uncertainty on thermal parameters except for T_g with an uncertainty of ±1°C

Test	$\left\|\left(\frac{\partial L}{\partial k}\right)U_k\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial \alpha}\right)U_\alpha\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial T_g}\right)U_{T_g}\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial Q_h}\right)U_{Q_h}\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial Q_m}\right)U_{Q_m}\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial Q_a}\right)U_{Q_a}\right\|$ m (%)	$\left\|\left(\frac{\partial L}{\partial R_b}\right)U_{R_b}\right\|$ m (%)	Local uncertainty on L m (%)
Test 1A	2.8 (4.7)	1.0 (1.7)	3.0 (5.1)	4.8 (8.1)	0.7 (1.2)	0.0 (0)	3.1 (5.2)	7.1 (12.0)
Test 1B	3.7 (4.9)	1.2 (1.6)	4.1 (5.4)	6.1 (8.0)	0.9 (1.2)	0.1(0.2)	3.9 (5.1)	9.2 (12.1)
Test 2	580 (5.6)	155 (1.5)	1060 (10.3)	661 (6.4)	175 (1.7)	27 (0.3)	465 (4.5)	1473 (14.2)
Test 3	205 (4.7)	39.0 (0.9)	376 (8.6)	325 (7.4)	154 (3.5)	97 (2.2)	206 (4.7)	605 (13.8)
Test 4	205 (6.8)	60.8 (2.0)	134 (4.4)	170 (5.6)	41.1 (1.4)	111 (3.7)	123 (4.1)	348 (11.5)

5.3. Global uncertainty using Monte-Carlo simulations

5.3.1. Impact of thermal parameters (L2 method)

The impact of the 7 thermal parameters listed in Table 3 is analysed for all test cases. Each of the parameters are varied according to their uncertainty, assuming a normal distribution and standard deviations equal to half of each uncertainty. A Sobol sequence is again utilized to generate 1024 samples of the 7 parameters for each of the test cases. The required borehole length evaluated using the L2 method of Eqs. (1)-(2), is calculated for each of the generated samples.

Figure 2 shows the distributions of the required borehole length H for all tests. The mean required borehole length is close to the ones reported in Table 4 for the base cases. The standard deviations vary from σ_H = 3.64 m for Test 1A to σ_H = 6.91 m for Test 4. The coefficient of variation (i.e. the standard deviation divided by the mean), however, is similar for all tests, from 5.7% for Test 4 to 7.2% for Test 2.

The uncertainty on the total required borehole length L can be evaluated from the distributions shown on Figure 2. For a confidence interval of 95%, the uncertainty is evaluated as $U_L = 2\sigma_L = 2\sigma_H \cdot nb$. The uncertainty evaluated from the present analysis is compared to the local propagation of uncertainties in section 5.3.3.

5.3.2. Impact of thermal parameters (L4 method)

Monte-Carlo simulations are conducted for the L4 method presented in Eqs. (4)-(5). The 7 thermal parameters are sampled in the same manner as in section 5.3.1 for the L2 method. Since the L4 method relies on hourly ground loads, preprocessing is necessary to include peak, monthly and annual load adjustments in simulations. A simulation is first conducted to identify the time index of the peak n_{peak} (corresponding to the time $t_{n_{peak}}$ at which any of the temperature constraints is active), using the design length of the L4 method in Table 4 and the base values of the parameters from Table 1. During Monte-Carlo simulations, ground loads preceding and including the peak are weighted proportionally to the sampled values of the peak, monthly and annual loads:

$$q_i = \begin{cases} q_{i,base} \cdot Q_h/Q_{h,base} & for\ t_{n_{peak}} - 0.25\ days < t_i \leq t_{n_{peak}} \\ q_{i,base} \cdot Q_m/Q_{m,base} & for\ t_{n_{peak}} - 30.25\ days < t_i \leq t_{n_{peak}} - 0.25\ days \\ q_{i,base} \cdot Q_y/Q_{y,base} & for\ t_i \leq t_{n_{peak}} - 30.25\ days \end{cases} \quad (8)$$

where $Q_{h,base}$, $Q_{m,base}$ and $Q_{y,base}$ are the base ground load values from Table 1 and $q_{i,base}$ are the hourly ground loads from Ahmadfard and Bernier [14]. The time intervals correspond to those used in the ASHRAE sizing equation presented in Eqs. (1)-(2). The analysis is restricted to Tests 3 and 4 since Tests 1A, 1B and 2 use synthetic ground loads.

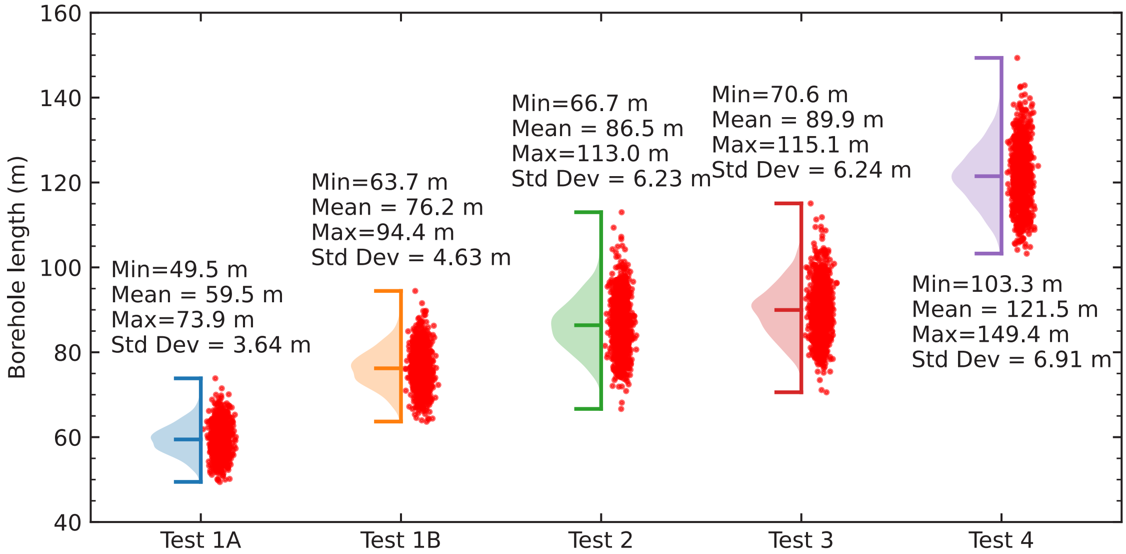

Figure 2. Distributions of the required borehole length H for all tests due to the uncertainty on the thermal parameters (L2 method)

Figure 3 compares the distributions of the required borehole length H for Tests 3 and 4 using the L2 and L4 methods. Different mean values are observed for Test 3. This is due to the peak demand being encountered in the first simulation month. In the L2 method, the total required length is evaluated at the end of the 10-year period, and the annual load imbalance thus lowers the total required length. The mean required lengths for Test 4 are very close, with only a 1 m difference. The standard deviations are however different. The L2 method seems to overestimate the standard deviation, and thereby the uncertainty on the total required length.

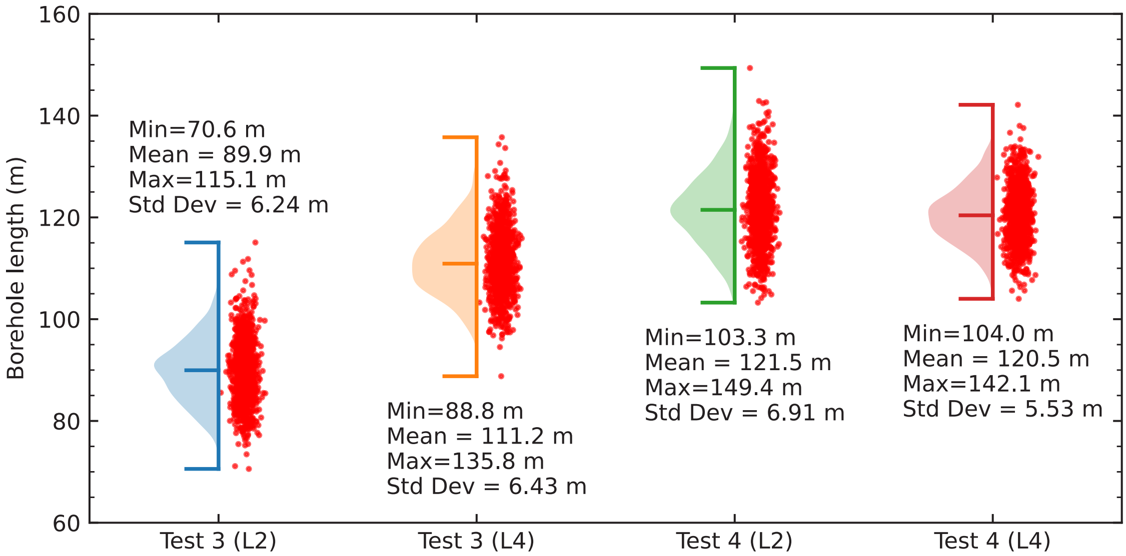

Figure 3. Distributions of the required borehole length H for Tests 3 and 4 due to the uncertainty on the thermal parameters using L2 and L4 methods

5.3.3. Summary of results on uncertainties

Table 6 presents a summary of results presented thus far. The local uncertainty on the total required borehole length evaluated from the propagation of local uncertainties using L2 methods is very close to the global uncertainty from the Monte-Carlo simulations for all tests using either L2 or L4 methods. This suggests that the local uncertainty provides an appropriate estimate of the overall uncertainty on the total required borehole length without having to resort to more involved calculations using Monte-Carlo simulations. Global uncertainties obtained with a L4 method (last column in Table 6) tend to be lower than the ones obtained using a L2 method (third column in Table 6). The average of the 12 uncertainty values presented in Table 6 is ±12.4% when assuming a ±10% uncertainty on thermal parameters except for T_g with an uncertainty of ±1°C (Table 3).

Table 6. Comparison of the uncertainty on the total required borehole length for three methods

Test	Local uncertainty L2 method m (%)	Global ($2\sigma_L$) L2 method m (%)	Global ($2\sigma_L$) L4 method m (%)
Test 1A	7.1 (12.0)	7.3 (12.2)	-
Test 1B	9.2 (12.1)	9.3 (12.2)	-
Test 2	1473 (14.2)	1495 (14.4)	-
Test 3	605 (13.8)	611 (13.9)	630 (11.6)
Test 4	348 (11.5)	346 (11.4)	277 (9.2)

5.4. Impact on the fluid temperature

The sizing equations can also be used to evaluate outlet fluid temperatures for a fixed borehole length. A borehole length H = 120.3 m is used for Test 4, as reported in Table 4 and evaluated using the L4 method. The outlet fluid temperature is predicted using Eq. (4), using the same sampling method as in sections 5.3.1 and 5.3.2 for the 7 thermal parameters. Figure 4 shows, on the left-hand side, simulated outlet fluid temperatures during the two-day period leading to the peak demand. The solid line corresponds to the mean value of the outlet fluid temperature across samples, and the highlighted area corresponds to the variation range of the outlet fluid temperature. The dashed line represents the maximum allowable outlet fluid temperature of 38°C. On the right-hand side, Figure 4 shows the distributions of the maximum outlet fluid temperature evaluated using the L2 (Eq. (3)) and L4 methods. The L2 method shows a mean maximum outlet fluid temperature (38.2°C) slightly above the maximum allowable value. This is in line with the evaluation of the required borehole length of 121.3 m reported in Table 4, which is higher than the length of 120.3 m used for the analysis and thus increases the temperature variations. As in the case of the distributions of the required borehole length shown on Figure 3, the standard deviation for the outlet fluid temperature using the L2 method overestimates the standard deviation evaluated using the L4 method. Using the standard deviation of the L4 method (= 1.06°C), the uncertainty on the outlet fluid temperature is of the order of ±2°C for Test 4.

The impact of uncertain inlet fluid temperatures on the heat pumps is problem dependent. For instance, the capacity of the heat pumps might be marginally impacted by a change in inlet fluid temperature which in turn might or might not significantly affect the thermal comfort of occupants. The impact of uncertain inlet fluid temperatures should therefore be assessed together with the control strategy and the building dynamics to conclude on possible negative impacts of design uncertainties.

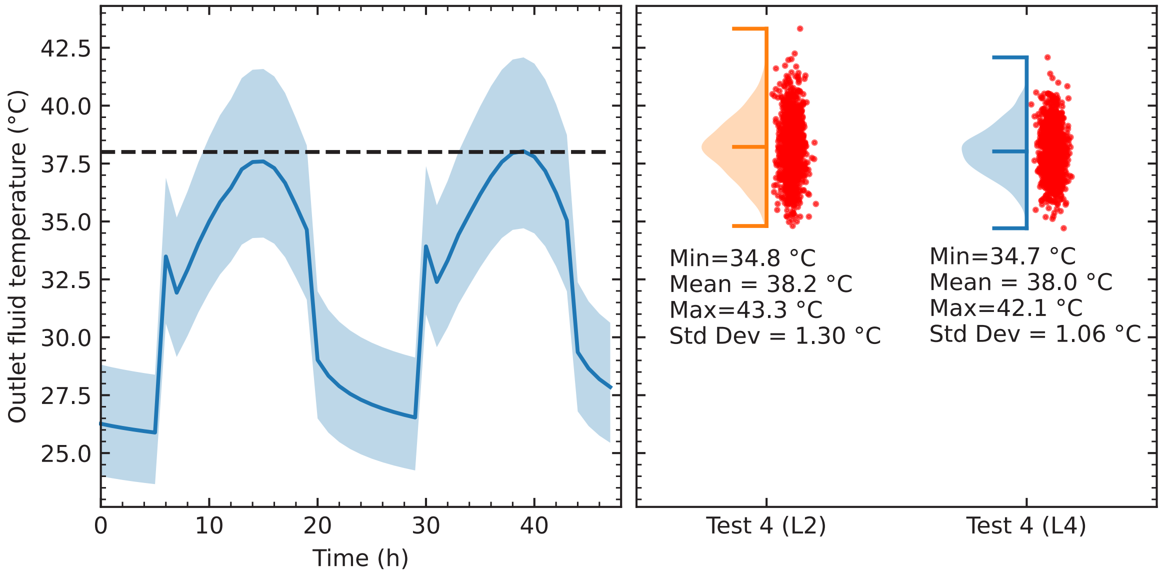

Figure 4. Outlet fluid temperature during a two-day period leading to the peak demand in Test 4 (left), and distributions of the maximum outlet fluid temperature due to the uncertainty on the thermal parameters using L2 and L4 methods (right)

6. Conclusion

Two methodologies are examined in this paper to evaluate the uncertainty on the design length of vertical ground heat exchangers. The first method calculates the local uncertainty by combining the uncertainty of each important parameter using sensitivity coefficients. The second method uses a Monte-Carlo type of simulation to obtain a global uncertainty. These methods are applied to so-called L2 and L4 dimensioning methods which are based on the three-pulse ASHRAE equation and on hourly simulations, respectively.

In the first part of this study, 14 calculation methods to determine the design length of vertical ground heat exchangers are reviewed and compared to each other using five test cases representing a broad range of conditions. This comparison indicates that one can expect a ±10% accuracy on calculation methods, independently of the uncertainty on parameters. The L2 and L4 calculation methods which are used in this paper are within this level of accuracy and could therefore be used with confidence for the reminder of the study.

The uncertainty analysis used five test cases with the L2 and L4 calculation methods with assumed uncertainties of ±10% on the ground thermal conductivity and diffusivity, peak ground load, average monthly ground load, annual ground load imbalance, equivalent borehole thermal resistance and ±1°C on the undisturbed ground temperature,

The local uncertainty varies from ±11.5% to ±14.2% on the determination of the design length for the five test cases. The contributions of each parameter vary across tests, and there is no clear trend as to which of the parameters have the single most and least significant impact on the overall uncertainty.

Global uncertainty calculations using Monte-Carlo simulations were performed with 1024 samples of 7 parameters, excluding the borehole positions, for each of the test cases. Results using a L2 method are very similar (from ±11.4% to ±14.4%) to the ones using the local uncertainty method for each test case. This suggests that the local uncertainty provides an appropriate estimate of the overall uncertainty on the total required borehole length without having to resort to more involved calculations using Monte-Carlo simulations. A step-by-step procedure to determine the local uncertainty is given in the appendix.

Finally, the variation of the borehole outlet fluid temperature is evaluated by varying the 7 thermal parameters for a fixed length. Using the standard deviation of the results of the L4 method (= 1.06°C), the uncertainty on the outlet fluid temperature is of the order of ±2°C at peak conditions.

7. Nomenclature

B: Spacing between boreholes (m)
c_p: Fluid specific heat capacity (J/kg·K)
H: Borehole length (m)
L: Total borehole length (m)
g: g-Function
k: Ground thermal conductivity (W/m·K)
$\dot{m}$: Fluid mass flow rate (kg/s)
nb: Number of boreholes
Q_h, Q_m, Q_a: Peak, monthly average, and annual average ground loads (W)
$Q_{h,base}$, $Q_{m,base}$, $Q_{a,base}$: Loads from Ahmadfard and Bernier [14] (W)
q_i: Hourly ground load per unit borehole length (W/m)
R_b: Borehole thermal resistance (m·K/W)
$R_{h,g}$, $R_{m,g}$, $R_{a,g}$: Ground thermal resistances for the peak, monthly, and annual loads (m·K/W)
T_g: Undisturbed ground temperature (°C)
T_{in}, T_{out}: Inlet and outlet fluid temperatures (°C)
$T_{out,min}$, $T_{out,max}$: Minimum and maximum allowable outlet fluid temperatures (°C)
t_i: Time (s)
U_{x_i}: Uncertainty on a variable x_i
α: Ground thermal diffusivity (m^2/s)
σ_{x_i}: Standard deviation on a variable x_i

8. Appendix – Procedure to obtain the local uncertainty

For designers who want to estimate the uncertainty in their design length, it is recommend to use the local uncertainty approach with the L2 method (Eqs. (1) and (6)). This involves the following steps. First, determine the peak ground load (Q_h), the average monthly ground load (Q_m) when Q_h occurs, and the annual ground load imbalance in the ground (Q_a). Second, obtain the ground thermal conductivity (k), the ground thermal diffusivity (α), and the undisturbed ground temperature (T_g) and calculate (or measure using a thermal response test) the borehole thermal resistance (R_b). With known values of T_{in} and T_{out} calculate the required borehole length using Eq. (1). The resulting value is L_{xi} in Eq. (7). Then, the designer should assign uncertainties (U_{x_i} in Eq. (6)) to the parameters. For example, if a $\pm 10\ \%$ uncertainty is assumed on a k value of 2.0 W/m·K, then U_{x_i} = 0.2 W/m·K. The next step involves the recalculation of Eq. (1) by varying each parameter by a small quantity Δx_i (1% of the actual value is a good value). Recalculate the borehole length with $x_i + \Delta x_i$, to get $L_{x_{i+\Delta x_i}}$. With this value evaluate Eq. (7). Repeat this process for every x_i and then evaluate the local uncertainty U_L using Eq. (6).

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