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Theoretical model for the thermal response of slender geothermal boreholes during short-term transient operating conditions

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Abstract

Many aspects of the operation of geothermal heating, ventilation, and air conditioning systems must be considered when designing and sizing geothermal heat exchangers. One of the most restrictive is the allowable temperature range of the heat carrying liquid, which limits the amount of heat that can be exchanged with the ground during short-term transient operating conditions. During such short-term events, thermal inertia of the heat carrying liquid, the grout within the borehole, and the surrounding ground plays an important role. Neglecting it during the design phase leads to overly conservative solutions for the geothermal heat exchanger, unnecessarily increasing the system's initial investment costs. Numerous theoretical models in the literature already account for that thermal inertia. Most of them, however, rely on substantial simplifications in the borehole geometry and/or its thermal characteristics. Only a few models, such as the Enhanced Multipole Method recently developed by the lead author, retain the full geometric and thermal complexity of the problem, paving the way for a new generation of high-fidelity models. In this work, the Enhanced Multipole Method, which only addresses the heat transfer problem in a two-dimensional plane perpendicular to the borehole, is extended in the axial direction to also include the thermal inertia and convective transport of the heat carrying liquid. The resulting model is benchmarked against detailed numerical simulations of the transient thermal response of a complete geothermal borehole, demonstrating both its accuracy and potential.

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Keywords: Geothermal heat exchangers; Geothermal boreholes; Transient operation; Thermal inertia; Exact theoretical model

1. Introduction

Mankind employs 23% of its final energy consumption in the heating and cooling of buildings [1]. This explains why replacing fossil fuel-based heating, ventilation, and air conditioning (HVAC) systems by more environmentally friendly solutions is of utmost importance nowadays. Geothermal-based HVAC systems, which harness the low enthalpy geothermal energy present beneath buildings, represent an excellent solution for that. Such a system consists in an electricity-driven water-to-water heat pump connected to a geothermal heat exchanger composed of multiple geothermal boreholes. Each borehole, like the one shown in Fig. 1, is equipped with one or more coaxial or U-shaped probes through which a heat carrying liquid flows to exchange heat with the ground.

To ensure the economic viability and environmental sustainability of geothermal HVAC systems, accurate predictions of their thermal performance over the building's lifespan are necessary at the design stages of the installation. These predictions rely on models that solve the governing heat transfer equations of the problem.

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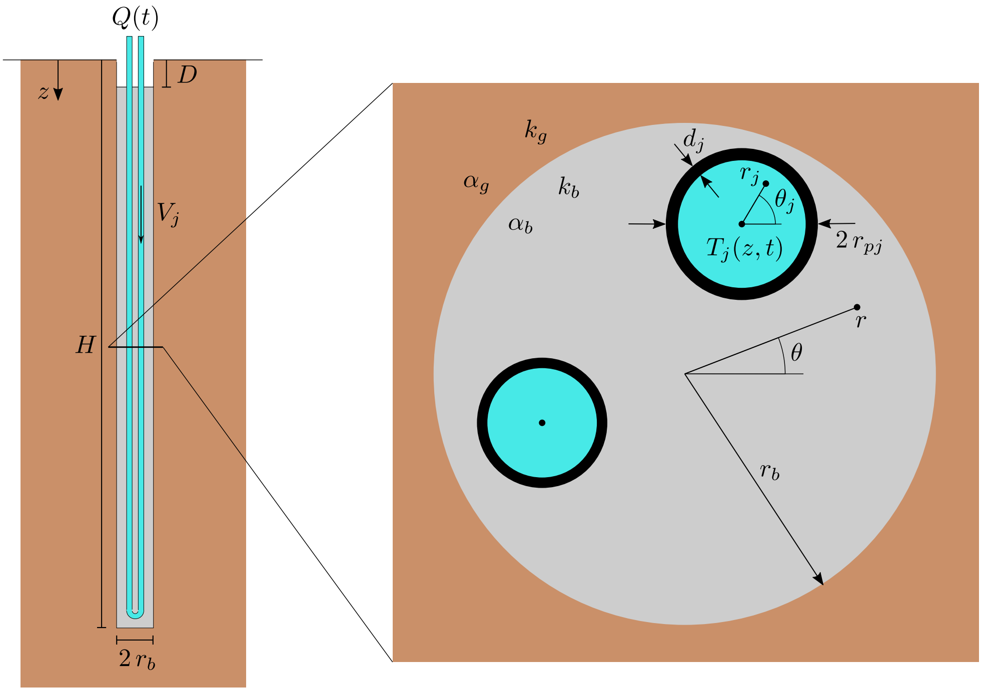

Figure 1. Sketch of a typical geothermal borehole equipped with a single U-shaped probe (black) through which a heat carrying liquid (cyan) flows to exchange heat with the ground (brown). The empty space between probe and ground is filled with grout (gray).

An important outcome are the maximum and minimum temperatures attained by the heat carrying liquid. To accurately forecast them, the model to use must account for thermal inertia of the heat carrying liquid, of the grout filling the borehole, and of the ground located close to the borehole. Otherwise, the thermal response to rapidly varying heating and cooling needs of the building is not reproduced correctly.

Recently, the Enhanced Multipole Method was developed by the first author to account for thermal inertia in the grout and in the ground surrounding the borehole [2-5]. Its advantage over other theoretical models is that no relevant simplifications are made to the geometrical configuration of the borehole or to its thermal behavior. Additionally, it converges on a term-by-term basis to the classical multipole method [6-8] in the limit of vanishing thermal inertia. Hence, it can be regarded as the seamless extension of the classical multipole method to problems with relevant thermal inertia in grout and ground.

The Enhanced Multipole Method, however, does not account for thermal inertia of the heat carrying liquid and only delivers the thermal response of grout and ground in two-dimensional planes perpendicular to the borehole. To overcome these two limitations and derive a transient model for a complete geothermal borehole, the authors have extended the problem in the axial direction of the borehole and included thermal inertia of the heat carrying liquid as well as its convective transport along the pipes. The aim of the present work is to give an overview of the capabilities of the resulting theoretical model, whose complete details can be found in [9].

2. Formulation of the problem

Geothermal boreholes present large disparities in time and length scales [10]. These disparities allow the governing equations to be simplified, enabling the development of fast and accurate models for the thermal response of geothermal boreholes. One such simplification, motivated by the extreme slenderness of real-world geothermal boreholes, is the neglection of axial heat conduction in grout and ground [10]. This allows the unsteady three-dimensional heat conduction problem in grout and ground to be replaced by two-dimensional heat conduction problems in planes perpendicular to the borehole. For the here-considered short-term transient operating conditions, these planes are only coupled to each other through the convective transport of heat along the pipes [9]. Therefore, formulation of the problem is split in the same two constituents, namely, the unsteady heat conduction in grout and ground and the unsteady convective transport of heat along the pipes.

2.1. Thermal response of grout and ground

Unsteady heat conduction in grout and ground takes place in independent two-dimensional planes perpendicular to the borehole. Fig. 1 represents one such plane. The borehole, of radius r_b, is filled with grout whose thermal conductivity k_b and thermal diffusivity α_b differ from the corresponding values k_g and α_g of the ground. Each pipe j inside the borehole is defined by its outer pipe radius r_{pj}, its wall thickness d_j, its thermal conductivity k_j, and its location in the polar coordinate system (r, θ) centered at the borehole. Additional polar coordinate systems (r_j, θ_j) centered at each pipe j are introduced as well for the formulation of boundary conditions at the pipes.

The conservation equation governing the unsteady heat conduction in grout and ground prescribes how the grout/ground temperature T evolves spatially and with time t:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}, \tag{1}$$

where the thermal diffusivity α is equal to α_b inside the borehole, where $r < r_b$, and equal to α_g outside the borehole, where $r > r_b$. Temperatures and heat fluxes in grout and ground need to be continuous at the borehole wall, located at $r = r_b$. Therefore, the previous conservation equation needs to be solved with the following continuity conditions at the borehole wall:

$$T|_{r=r_b^-} = T|_{r=r_b^+} \qquad and \qquad -k_b\frac{\partial T}{\partial r}\bigg|_{r=r_b^-} = -k_g\frac{\partial T}{\partial r}\bigg|_{r=r_b^+}. \tag{2}$$

Far from the borehole, the ground temperature must tend to the unperturbed ground temperature T_∞. Due to the presence of a geothermal heat flux, that unperturbed temperature is a function of the vertical coordinate z so that the boundary condition to enforce far from the borehole changes with the depth of the considered two-dimensional plane:

$$r \to \infty\colon\ T \to T_\infty(z). \tag{3}$$

Also the temperature T_j of the heat carrying liquid inside each pipe j changes with depth, triggering different thermal responses for grout and ground at each position z along the borehole. This influence is exerted through the boundary condition enforced at the outer wall of each pipe j [10]:

$$-k_b r_{pj}\frac{\partial T}{\partial r_j}\bigg|_{r_j=r_{pj}} = \frac{T_j - T|_{r_j=r_{pj}}}{R_{pj}}, \tag{4}$$

where the pipe's inner thermal resistance R_{pj} accounts for the quasi-steady heat conduction inside the thin pipe wall and for the turbulent transport of heat inside the heat carrying liquid. Modelling the latter phenomenon with a convective heat transfer coefficient h_j leads to the following expression for R_{pj} [10]:

$$R_{pj} = \frac{1}{(r_{pj} - d_j)h_j} + \frac{1}{k_j}\ln\left(\frac{r_{pj}}{r_{pj} - d_j}\right). \tag{5}$$

Finally, the unperturbed ground temperature is imposed as initial condition everywhere in the problem:

$$t = 0\colon\ T = T_\infty(z). \tag{6}$$

2.2. Thermal response of heat carrying liquid

The transport of heat along each pipe j is described using the following conservation equation, derived from applying the energy conservation equation in integral form to an infinitesimal slice of pipe [9]:

$$\rho c A_j\frac{\partial T_j}{\partial t} + \rho c A_j V_j\frac{\partial T_j}{\partial z} = -q_j. \tag{7}$$

The first term on the left-hand side represents thermal inertia of the liquid, with ρ being the density of the liquid, c its specific heat capacity, and A_j the inner cross section of pipe j. The second term on the left-hand side is the convective transport of heat, where V_j represents the bulk velocity of the liquid. Finally, the right-hand side is the heat injection rate per unit pipe length, q_j, exchanged with the grout surrounding the pipe. It is obtained by integrating the heat flux along the outer wall of pipe j [10]:

$$q_j = \int_{-\pi}^{\pi} -k_b r_{pj} \frac{\partial T}{\partial r_j}\bigg|_{r_j=r_{pj}} \mathrm{d}\theta_j \,. \tag{8}$$

To complete the formulation of the problem, initial and boundary conditions are required for the fluid temperatures T_j. As initial condition, the unperturbed ground temperature T_∞ is imposed on all pipes:

$$t = 0\colon\ T_j = T_\infty(z). \tag{9}$$

The boundary condition to enforce at the inlet of each pipe j, however, depends on the specific pipe configuration of the borehole. For the case depicted in Fig. 1, in which the borehole is equipped with a single U-shaped probe, the following two conditions are formulated:

$$z = D\colon\ \dot{m}c(T_1 - T_2) = Q(t) \qquad \text{and} \qquad z = H\colon\ T_1 = T_2. \tag{10}$$

The first condition, imposed at the buried depth D of the borehole, ensures the prescribed heat injection rate $Q(t)$ is always satisfied, with the mass flow rate circulating through the U-shaped probe being given by

$$\dot{m} = \rho A_1|V_1| = \rho A_2|V_2|. \tag{11}$$

At the bottom of the borehole, located at a depth $z = H,$ the second condition ensures energy conservation between the two pipes by imposing equal fluid temperatures there.

2.3. Laplace transformation

The problem to solve is transformed to the complex-valued Laplace plane to handle its time dependency [11]. Most equations remain thereby unaltered, with just temperatures, heat injection rates per unit pipe length, and heat injection rate bearing now a tilde (~) as superscript to denote their dependence on position *s* in the complex-valued Laplace plane instead of on time *t*. Only the energy conservation equation in grout and ground,

$$\frac{s}{\alpha}\left[\tilde{T} - \frac{T_\infty}{s}\right] = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{T}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \tilde{T}}{\partial \theta^2}, \tag{12}$$

the energy conservation equation in each pipe j,

$$\rho c A_j s\left[\tilde{T}_j - \frac{T_\infty}{s}\right] + \rho c A_j V_j \frac{\partial \tilde{T}_j}{\partial z} = -\tilde{q}_j, \tag{13}$$

and the boundary condition enforced far from the borehole,

$$r \to \infty\colon\ \tilde{T} \to \frac{T_\infty}{s}, \tag{14}$$

change with the application of the Laplace transform.

Once the exact mathematical solution to the formulated problem is known [9], time evolution of the variables is obtained from inverting the Laplace transformation. This is accomplished by using a numerical inversion algorithm that correctly handles the singularities of the functions in the complex-valued Laplace plane [9] and that implements convenient speedup strategies [12] to ensure overall low computational costs.

3. Solution procedure

To solve the problem formulated in the previous section, first the heat transfer problem in grout and ground is addressed. Then, the governing equations for the pipes are solved. This two-step approach is analogous to the one followed for the formulation of the problem. What follows next is a brief overview of the strategy followed in [9] to derive exact expressions for the temperatures $\tilde{T}_j$ of the heat carrying liquid in the pipes.

The thermal response of grout and ground is obtained using the Enhanced Multipole Method [2-5]. One of the results supplied by the method are expressions that relate the heat injection rates per unit pipe length $\tilde{q}_j$ with the fluid temperatures $\tilde{T}_j$ and the apparent temperature of the ground, which is the temperature at which the borehole perceives the surrounding ground [3]. These expressions involve thermal resistances and weighting coefficients that embody all the features and characteristics of the heat transfer problem in the two-dimensional planes perpendicular to the borehole, including the thermal inertia of grout and ground.

Combination of the expressions provided by the Enhanced Multipole Method with the governing equations for the pipes delivers a system of ordinary differential equations for the fluid temperatures $\tilde{T}_j$. The challenge does not lie in solving the system of equations, but in ending up with expressions for the fluid temperatures $\tilde{T}_j$ that perfectly blend into the existing state of the art. In this sense, the derivation of expressions that seamlessly fit into the theoretical framework being developed by the leading author since 2011 [10,13-17] represents a crucial achievement of the present work.

4. Numerical example

To demonstrate the capabilities of the developed model, the borehole configuration shown in Fig. 2 is considered. It consists in a borehole of diameter $2r_b = 150$ mm and length $H = 150$ m whose buried depth is $D = 0.5$ m. Two equal pipes are placed inside the borehole, forming a single U-shaped probe. Their outer diameters are $2r_{pj} = 40$ mm, their wall thicknesses equal to $d_j = 3.7$ mm and their centres, placed symmetrically with respect to the centre of the borehole, are separated a distance of 70 mm from each other. The pipes are made from HDPE with a thermal conductivity of $k_j = 0.42$ W/(m K).

The U-shaped probe is fed through Pipe 1 with $\dot{m} = 0.6$ kg/s of pure water as heat carrying liquid. Its density, specific heat capacity, thermal conductivity, and dynamic viscosity are equal to 999 kg/m^3, 4186 J/(kg K), 0.59 W/(m K), and $1.1 \cdot 10^{-3}$ kg/(m s), respectively. Using Gnielinski's correlations for the convective heat transfer coefficients h_j [18], the following values result for the inner thermal resistances of the pipes:

$$R_{pj} = 0.508 \frac{\mathrm{m\,K}}{\mathrm{W}}. \tag{15}$$

The ground surrounding the borehole presents a thermal conductivity of $k_g = 1.8$ W/(m K) and a thermal diffusivity of $\alpha_g = 7.83 \cdot 10^{-7}$ m^2/s, values that differ from the ones corresponding to the grout filling the borehole: $k_b = 1.73$ W/(m K) and $\alpha_b = 5.0 \cdot 10^{-7}$ m^2/s.

A time constant heat injection rate equal to $Q = 5250$ W is enforced onto the borehole from time $t = 0$ s onwards. The transfer of this heat to the ground alters the initially unperturbed temperature profile of the ground, characterized by a mean annual temperature, at the ground surface, of 15 °C and a geothermal heat flux of 0.06 W/m^2.

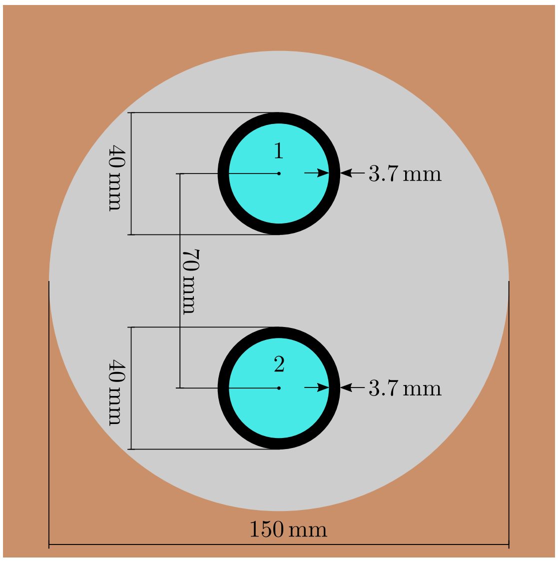

Figure 2. Cross section of the borehole configuration considered for the demonstration of the capabilities of the developed model for the short-term transient response of geothermal boreholes.

Fig. 3 shows the time evolution of the inlet and outlet temperatures, T_{in} and T_{out} respectively, of the considered borehole. Red curves correspond to results from the theoretical model developed by the authors in [9], black curves represent results from a detailed numerical simulation performed with the commercial software package COMSOL [19], and green curves show results from the g-function model [20] as implemented in the open-source Python library pygfunction [21,22].

The most relevant conclusion of the plotted curves is the almost perfect match between the developed theoretical model and the reference solution obtained with COMSOL. In fact, the small differences observed are mainly caused by the finiteness of the employed spatial and temporal meshes in COMSOL as these differences can be further reduced by further refining the meshes. This outstanding accuracy of the developed model can be explained by the limited relevance three dimensional effects have in the unsteady heat conduction problem in grout and ground, especially during the short-term transient operating conditions considered here.

Comparing the results of the developed theoretical model with the temperatures predicted by the g-function model, it becomes evident that thermal inertia plays a fundamental role in the thermal response to short-term transient conditions. In fact, the g-function model systematically overpredicts the real temperatures in the borehole, leading to oversized geothermal heat exchangers that have a negative impact onto the initial investment costs of the system.

The developed theoretical model also allows the study of the time evolution of the fluid temperatures along the complete pipes. To showcase this, Fig. 4 represents the time evolution of the fluid temperature $T_1(z,t)$ along Pipe 1. The inlet temperature plotted in Fig. 3 corresponds in this figure to the fluid temperature attained at the buried depth D of the borehole: $T_{in}(t) = T_1(D,t)$. During the first 208.7 seconds of the problem, the initial temperature distribution inside Pipe 1, equal to $T_\infty(z)$, is washed out to Pipe 2 through the bottom of the borehole and replaced by hotter heat carrying liquid entering Pipe 1 through the top of the borehole. Since initially Pipe 2 is also at the non-uniform temperature $T_\infty(z)$, the temperature at the entrance of Pipe 1 required to ensure the prescribed constant heat injection rate Q at all times also changes over time, leading to the temperature oscillations observed in Fig. 4 as well as in Fig. 3.

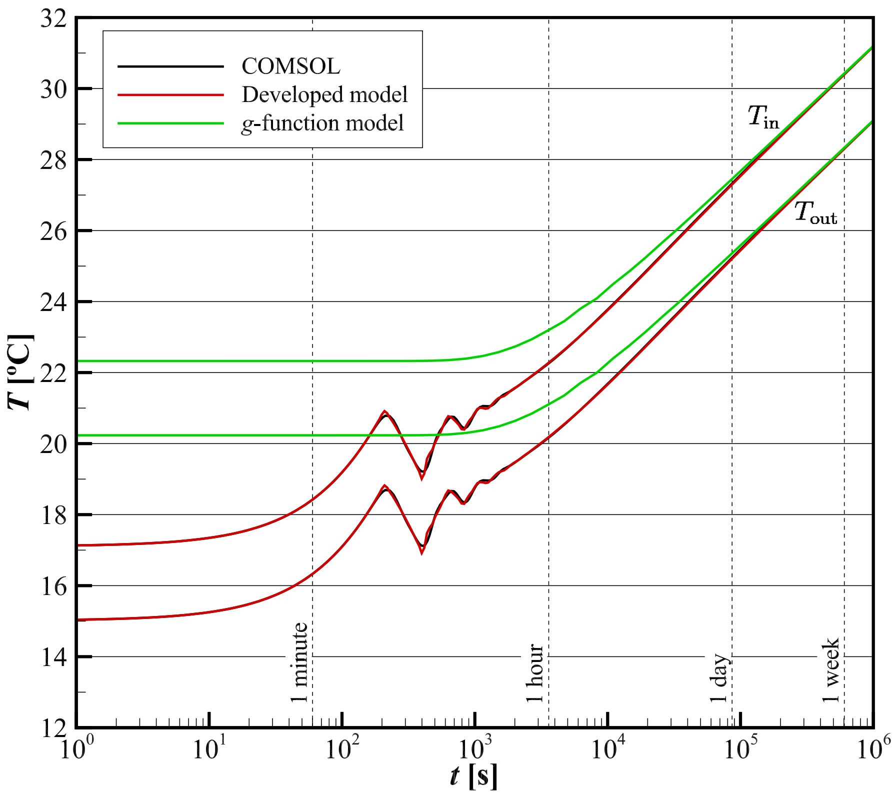

Figure 3. Time evolution of the inlet and outlet temperatures, T_{in} and T_{out} respectively, of the considered borehole. Results for three different models are presented: a detailed numerical simulation performed with COMSOL (black lines), the results obtained with the developed theoretical model (red lines) and results from the g-function model computed using the library pygfunction (green lines).

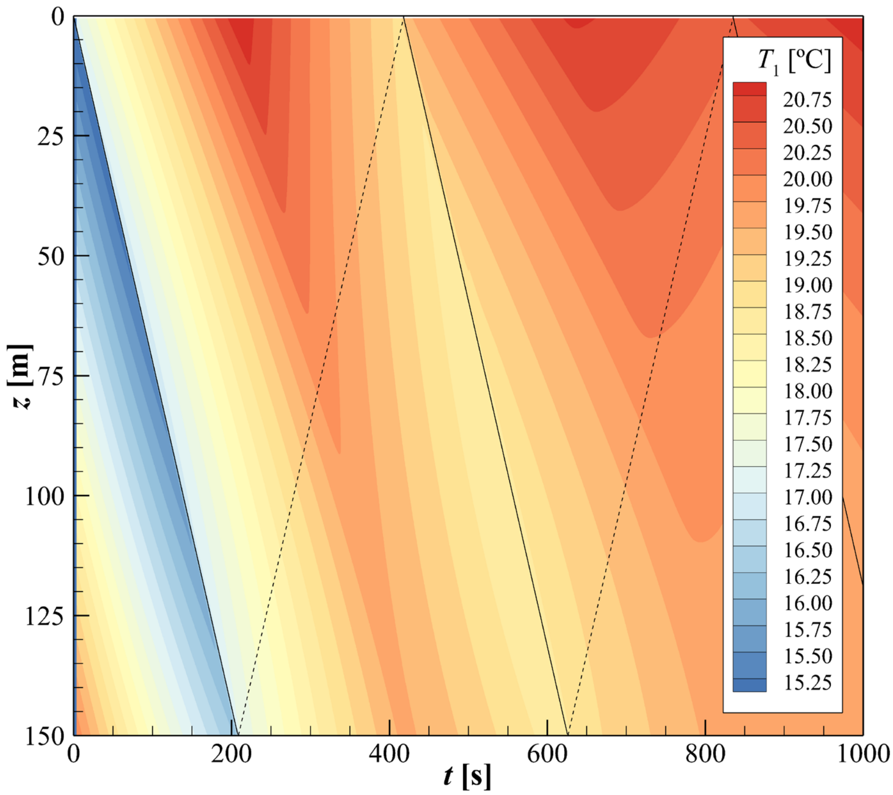

Figure 4. Time evolution of the fluid temperature $T_1(z,t)$ along the complete Pipe 1 of the considered borehole.

5. Conclusions

For rapidly varying heating and cooling needs in buildings, thermal inertia of the heat carrying liquid, of the grout filling the boreholes, and of the ground located close to the boreholes plays a fundamental role in the thermal response of geothermal heat exchangers. To correctly predict this response, a new theoretical model has been developed that accounts for the aforementioned thermal inertia without requiring relevant simplifications of the geometry and/or thermal characteristics of geothermal boreholes. Comparisons with detailed numerical simulations show the outstanding capabilities of the developed model, while comparisons with results from the *g*-function model highlight the importance of thermal inertia in the correct design and sizing of geothermal heat exchangers.

Acknowledgements

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