

# DEVELOPMENT AND TEST OF AN ADAPTIVE MODEL BASED PREDICTIVE CONTROLLER FOR SMALL HEAT PUMP HEATING SYSTEMS

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**Abstract:** The heat pump heating systems in single family houses typically are controlled with a bang-bang controller for the return flow temperature. This type of controller permits sufficient comfort for the residents, but generally it is unable to take into account the dependence of the heat pump's efficiency on the outdoor air temperature. Therefore the heat pump consumes more electrical energy and thus keeps the heating costs higher than necessary. The Model Predictive Control (MPC) design technique is chosen to optimize the energy consumption and the comfort taking into account weather forecast data. To adjust the parameters of the MPC Controller an identification algorithm is implemented. Its function is to automatically detect the thermal behavior of the house and its heating system by on-line parameter identification techniques in order to adjust the controller parameters according to preset demands.

**Key Words:** *heat pumps, parameter estimation, weather forecasts, predictive controller*

## 1 INTRODUCTION

Small heat pump heating systems are controlled almost exclusively by bang-bang controllers whose setpoint is chosen as a function of the outdoor temperature. The main strategy of such controllers is to optimally control the heat delivery system. Special demands on the heat pump, such as extended running times, high portions of low-tariff heating, or low usage of auxiliary power, are given secondary attention only.

A new controller for heat pumps has been developed at the Measurement and Control Laboratory using the model predictive control concept. Contrary to conventional heat pump controller strategies, which exclusively use the actual measurements, the model predictive controller calculates the required heat energy by means of the predicted course of the outdoor temperature, the efficiency of the heat pump, the power costs (high and low tariff), and the power cut-off times imposed by electric power providers. To transform the required heat energy into an on/off switch signal for the heat pump, the pulse-width modulation technique is used. To solve the model predictive control problem for the specific problem of house heating, a dynamic linear model of the house is required. The model for the controller is based on simple thermodynamic laws. The resulting equations contain a set of a priori unknown parameters; furthermore, since the parameters of the model differ from one house to another, an identification mechanism is indispensable. Within the scope of this project a self-adapting MPC controller has been developed. Its function is to automatically detect the

thermal behavior of the house and its heating system by on-line parameter identification techniques in order to adjust the controller parameters according to present demands. Different families of recursive parameter identification methods are compared. Furthermore, the robustness with respect to disturbances, the detection of solar radiation, and the consistency of the identified parameters are investigated. The identified model must be able to predict reliably the house dynamics for the next day.

## 2 MODEL PREDICTIVE CONTROL FOR A HOUSE HEATING SYSTEM

To solve the model predictive control problem, a dynamic linear model of the house is required. Preceding studies have shown that a model of second order is able to describe the most important dynamics (Bianchi 2005; Shafai 2003; Shafai 1999; Wimmer 2001). Figure 1 shows the considered thermodynamic effects and heat flows of the house model of second order.

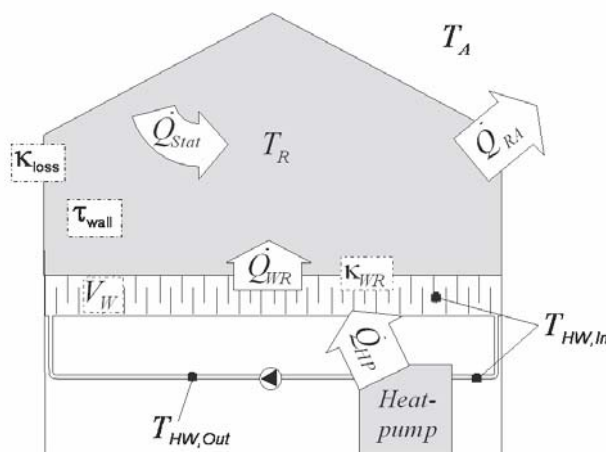


Figure 1: Thermodynamics of the house model of second order

Since the model for the controller is based on simple thermodynamic laws, the resulting differential equation system can be written as a state space model, as shown in Figure 2:

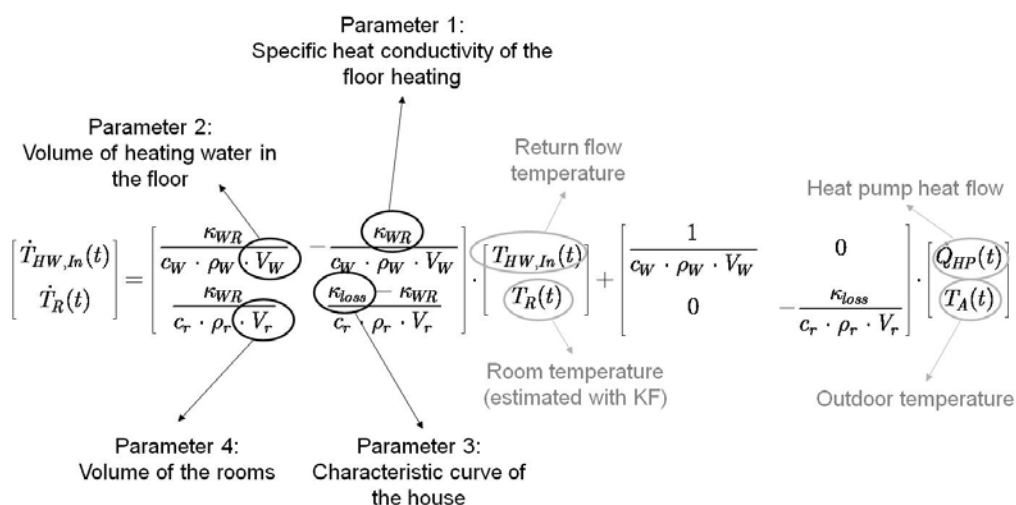


Figure 2: State-space second-order model of a house

The required measurement data are the outdoor temperature, the on/off state of the heat pump, and the return flow temperature. If it is not measured, the room temperature is estimated with a Kalman filter by means of the linear model.

Instead of the parameters  $V_r$ ,  $\rho_r$  and  $c_r$  the time constant of the house  $\tau_{wall}$  is used as fourth parameter. The heat pump control problem is solved with the model predictive control algorithm (MPC) and pulse-width modulation (PWM). The MPC controller calculates the optimal heat distribution over one day, taking into account the weather forecast, the dynamics of the house, as well as any low-tariff periods and power cut-off periods. A detailed description of this controller can be found in (Bianchi 2007; Bianchi 2005; Shafai 2003; Shafai 1999; Shafai 2002; Wimmer 2001).

### 3 PARAMETER ESTIMATION OF THE THERMAL BEHAVIOR OF A HOUSE

The resulting second-order house model, which is used by the MPC optimization, contains four a priori unknown parameters. The main idea of this project was to find and implement a robust identification algorithm to approximate the house parameters from the given measurements of the room, heating water and outdoor temperatures. The parameter identification procedure has to find an optimal parameter set for the building model, such that on one hand the error between the current and the estimated temperatures is minimal and on the other hand the thermal behavior of the building can be forecast correctly for a time horizon of typically one day.

Since the house model consists of a set of physical differential equations, a further condition is that the identified parameters should be physically justifiable. The system identification problem is formulated generally as a minimization problem, where the parameters to be found minimize a loss function (e.g. the sum of the square errors between measurement and simulation). The solution of the system identification can depend strongly on disturbances such as modeling errors (neglect of the solar radiation and/or the floor dynamics in the house model of second order) and measurement inaccuracies (measurement noise and approximation of the heat pump energy). It is therefore important that for the problem of the parameter estimation of the building the identification method is robust against these disturbances.

#### 3.1 Introduction to the identification methods investigated

Many approaches can be used to solve the system identification problem. For the specific case of this project (the second-order house model), parametric identification methods have been used. The identification algorithm had to satisfy the following conditions:

- Robustness in respect to disturbances such as measurement noise and modeling errors.
- Detection of the solar radiation and consideration of the mean gained energy in the model.
- Recursive implementation of the algorithm to permit the real-time identification of the parameters in an industrial heat-pump controller.
- The identified model must be able to anticipate reliably the house dynamics for the next day.

Three families of recursive parameter identification methods have been analyzed:

- **Pseudo-Linear Regression Methods:** They include the identification methods whose model structures can be expressed as a linear regression and thus may be solved with the least squares method (Ljung 1987; Ljung 1983).
- **Prediction Error Methods:** This family focuses on the minimization of a cost function of the error between measured and estimated signals (Figure 3). Since generally no analytic solution exists for the minimization problem, numerical search algorithms (such as the Gauss-Newton method) are used (Ljung 1983; Moore 1979). An example is the recursive maximum likelihood method (RML).

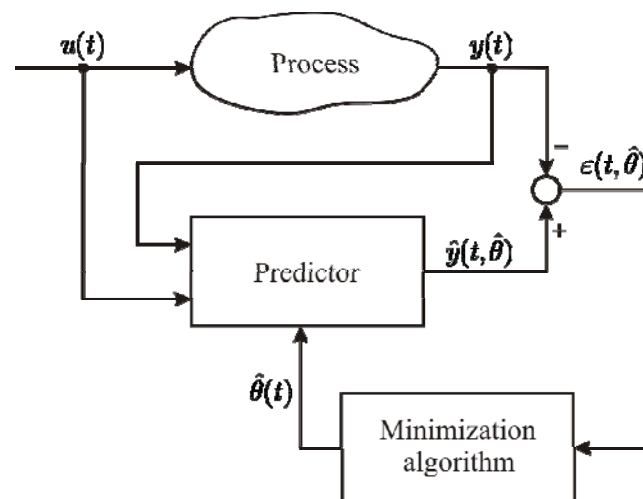


Figure 3: Prediction Error Methods for parameter identification

- **Bayesian Approach:** These methods model the parameters as random variables. Starting from an a priori density function of the parameter, while the a posteriori density function is estimated at each time step based on the new measurement data. The parameters are thus estimated evaluating the a posteriori density function (Goodwin 1977; Ljung 1983). An example is the extended Kalman filter (EKF) shown in Figure 4.

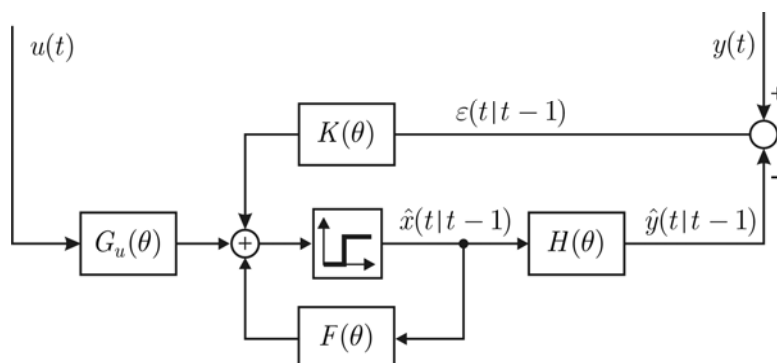


Figure 4: Kalman Filter

Results have shown that recursive and offline least-squares methods are inadequate for the task of parameter identification, since the estimated parameters are biased. Thus the identified model cannot reproduce the house dynamics and is often unstable.

Better results can be obtained with the family of the prediction error methods, because these are not restricted to the ARX model structures. In particular, the maximum likelihood method and the extended Kalman filter have produced the most accurate results.

### 3.2 Conditions for a robust parameter identification

The measurements of the temperatures and heat flows in the heating system make possible the estimation of the important characteristics of the building, which are required by the model predictive controller of the house heating with the heat pump. The optimal structure, the optimal cost function, and the required measurements for consistent parameter identification have been investigated. Different model structures have also been compared.

A further problem that has been examined is the influence of disturbances such as measuring noise or the solar radiation on the solution of the identification problem. Since these influences are not deterministic, the solution of the identification problem depends also on the selected model structure of the noise and on the choice of the cost function. For the comparison different predictors are examined, with a weighted quadratic or the logarithmic Maximum Likelihood cost function. The data of the reference building have been synthetically generated by a house model of third order with a slow-acting floor heating (large modeling error for the house model of second order). See also (Bianchi 2007).

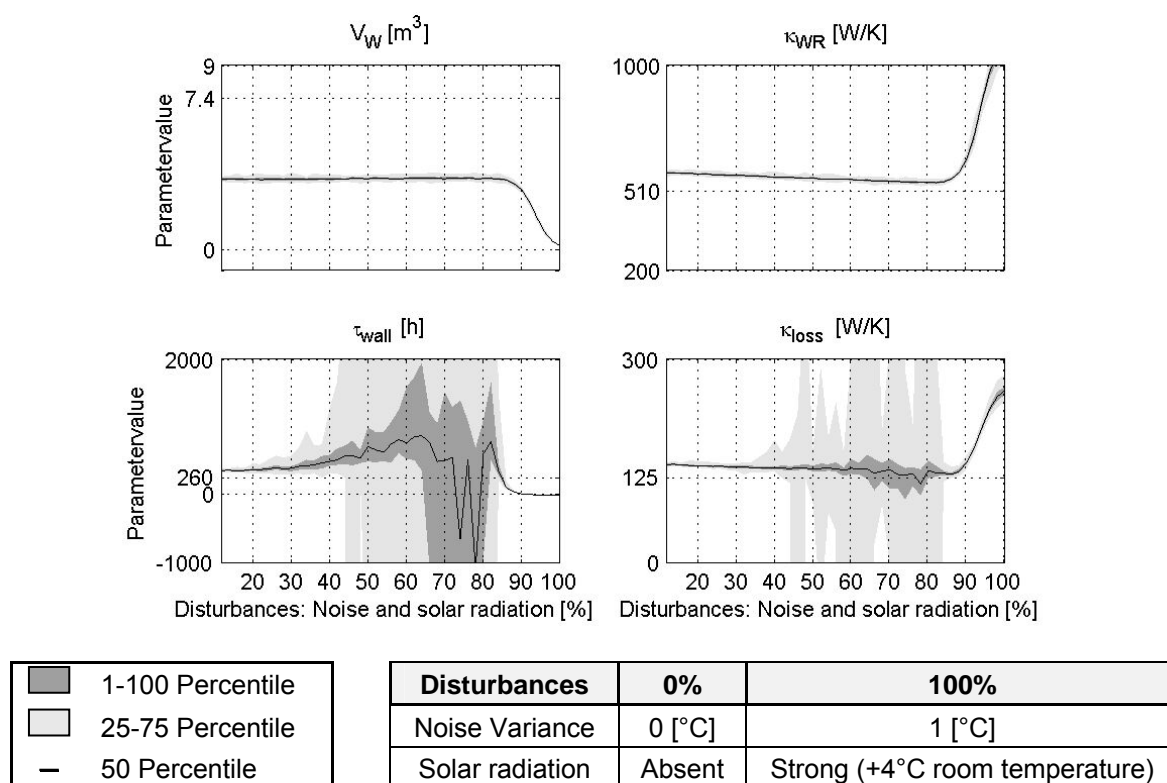
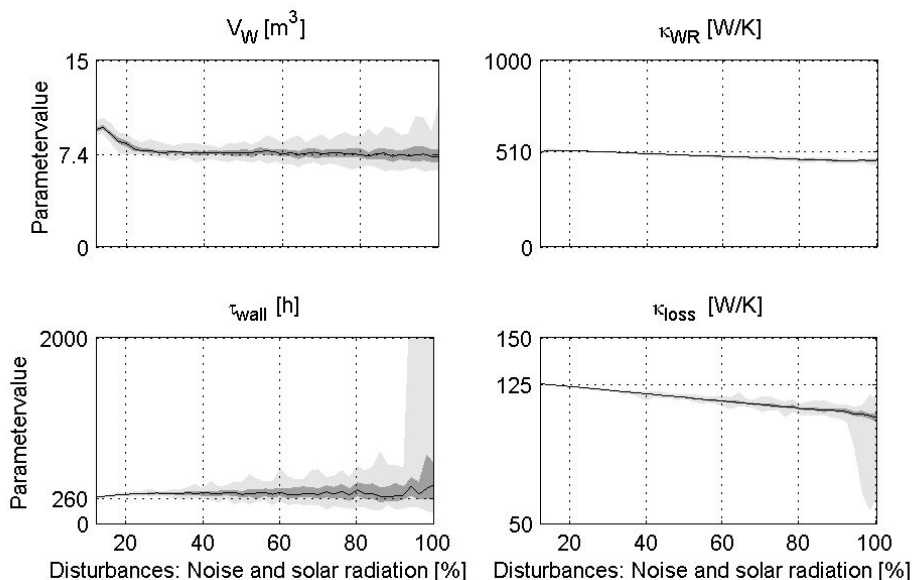


Figure 5: Solution of the LS problem for the house model of second order

The results show that the choice of an ARX structure, which is used to estimate the solution with standard least squares methods, is not suitable with the building model of second order, because the influence of the disturbances (solar radiation, modeling errors, noise) on the consistence of the parameter identification is too strong. The parameter values for the second-order house model which have been judged to be good approximations of the reference model are  $V_w = 7.4 \text{ [m}^3\text{]}$ ,  $\kappa_{WR} = 510 \text{ [W / K]}$ ,  $\tau_{wall} = 260 \text{ [h]}$  and  $\kappa_{loss} = 125 \text{ [W / K]}$  (dashed lines in Figure 6).



**Figure 6: Solution with an OE structure and a Maximum Likelihood cost function for the house model of second order**

Quite contrary to ARX structures, the OE structure with the logarithmic maximum likelihood cost function or the Kalman filter are very robust with respect to disturbances (Figure 6). Both methods produce similar results, but to solve the problem with a Kalman filter the knowledge of the covariance matrices of the noise are required.

The solar gain can also be detected (shown as a flattening of the heating curve). Without the measurement of the room temperature the detection of the solar gains is not possible, and due to the simple model structure the parameter identification is more difficult. Therefore, for robust parameter identification the measurement data of the outdoor, heating water and room temperatures are required.

### 3.3 Online methods for the parameter identification

Various recursive methods for the parameter identification have been investigated and compared. A brief explanation of the identification algorithms derived and of the results obtained is given in this subsection.

The least squares method is one of the simplest algorithms to implement. The computation of the parameters with a least squares method requires a transformation of the differential equation in Figure 2 into the following form:

$$y(t) = \Phi^T(t) \cdot \theta + e(t), \quad \text{Equation 1}$$

where  $y(t)$  is the output vector,  $\Phi(t)$  the regressor,  $\theta$  the parameter vector, and  $e(t)$  the noise (assumed as white). For the computation of the solution a weighted quadratic loss

function is used:

$$V_N(\theta) = \sum_{t=1}^N \varepsilon^T(t, \theta) \cdot \Lambda^{-1} \cdot \varepsilon(t, \theta). \quad \text{Equation 2}$$

The covariance matrix  $\Lambda$  of  $y(t)$  is the optimal choice for the weighting matrix. The signal  $\varepsilon(t, \theta)$  corresponds to the residuals. The recursive algorithm for the online parameter identification can be easily derived from the offline solution (Bianchi 2007).

Another approach for the parameter identification is the extended Kalman filter. This method assures that the covariance matrix of the state vector error of a linear differential equation is minimized:

$$\Sigma(t+1|t) = E \left\{ \left[ x(t+1) - \hat{x}(t+1|t) \right] \left[ x(t+1) - \hat{x}(t+1|t) \right]^T \right\}. \quad \text{Equation 3}$$

In the extended version of the Kalman filter the parameters are also modeled as states:

$$\begin{bmatrix} x(t+1) \\ \theta \end{bmatrix} = \begin{bmatrix} F(\theta) \cdot x(t) + G_u(\theta) \cdot u(t) \\ \theta \end{bmatrix}$$

$$y(t) = H(\theta) \cdot x(t). \quad \text{Equation 4}$$

As a consequence of this extension the linear equation in Figure 2 becomes nonlinear. To solve the Kalman filter problem a linearization of the extended state-space model is required at each time step (Bianchi 2007; Goodwin 1977).

A more generic family of identification methods are the PEM algorithms. The chosen loss function is often a quadratic sum of the residuals or the logarithm of the maximum likelihood function:

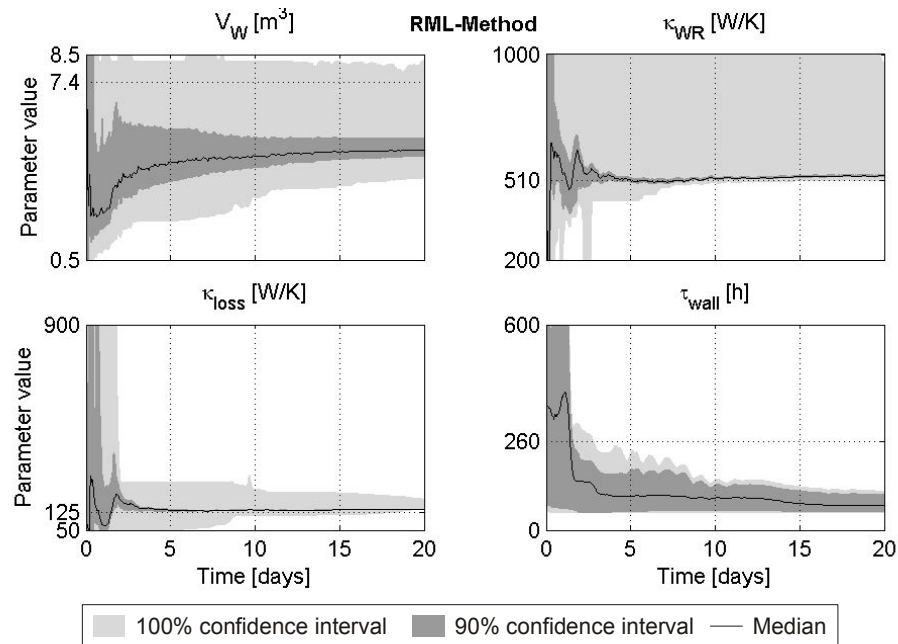
$$V_t(\theta) = -\frac{1}{2} \sum_{s=1}^t \left[ \varepsilon^T(s, \theta) \Lambda^{-1}(\theta) \varepsilon(s, \theta) - \ln \left[ \det(\Lambda(\theta)) \right] \right]. \quad \text{Equation 5}$$

To derive the recursive form of the parameter identification method, the Gauss-Newton algorithm is used:

$$\hat{\theta}(t) = \hat{\theta}(t-1) - \left[ V_t''(\hat{\theta}(t-1)) \right]^{-1} \cdot V_t'(\hat{\theta}(t-1)). \quad \text{Equation 6}$$

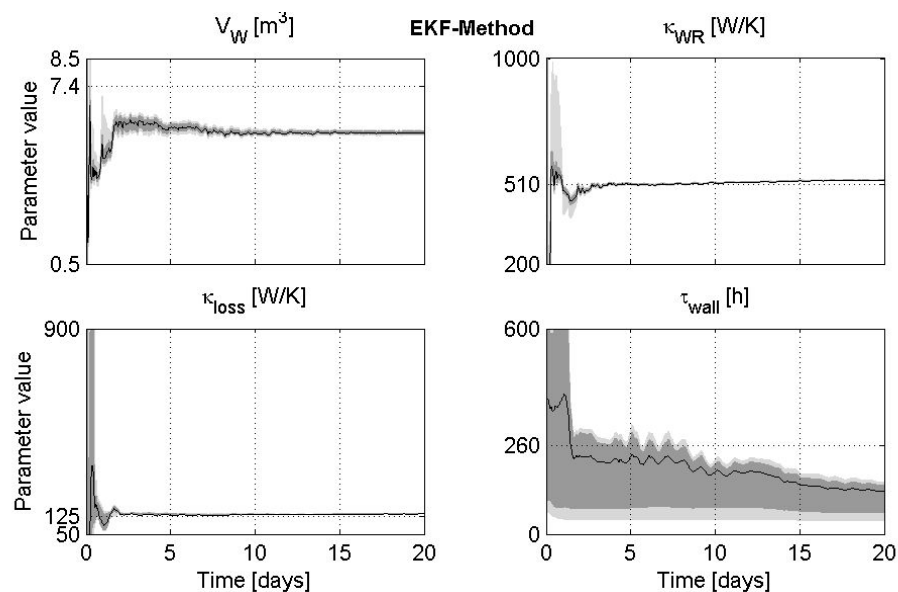
The derivatives of the loss function are calculated by means of the estimation model (Bianchi 2007; Goodwin 1977; Ljung 1987; Ljung 1983; Moore 1979; Söderström 1989).

A recursive least squares method (RLS), the extended Kalman filter as parameter identification method (EKF), the recursive prediction error method with a quadratic loss function (RPEM), the recursive maximum likelihood method (RML), and a modified RML method (MRML) with an EKF as estimator (Chu, 1996) have been implemented and compared. The results show that the EKF and MRML methods produce the best results.



**Figure 7: Results of the Monte Carlo simulations with the RML-method**

The parameters estimated by the RPEM and RML methods are good also, but the convergence is not as fast. However the estimation with the RLS method differs too much from the correct parameters and cannot be used for a daily forecast of the thermal behavior of the house.



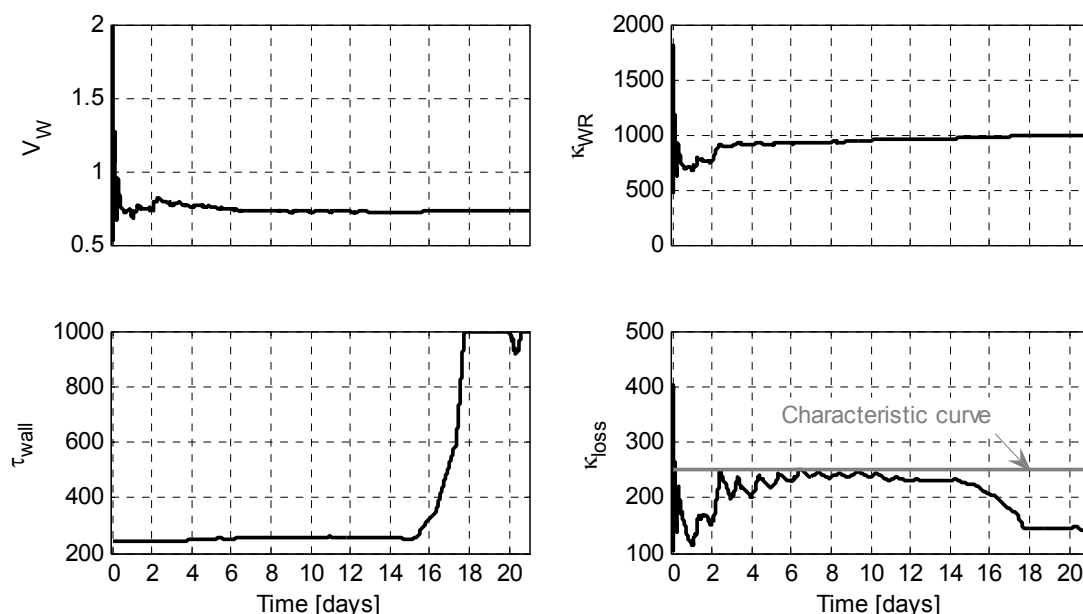
**Figure 8: Results of the Monte Carlo simulations with the EKF method**

#### 4 RESULTS AND CONCLUSION

The identification algorithms presented in Section 3.3 are adequate for an implementation in a real-time system. To avoid numerical problems, the stability and consistence of the identified model are verified at each time step. After that, the identified parameters are passed to the MPC controller to calculate the optimal heating.

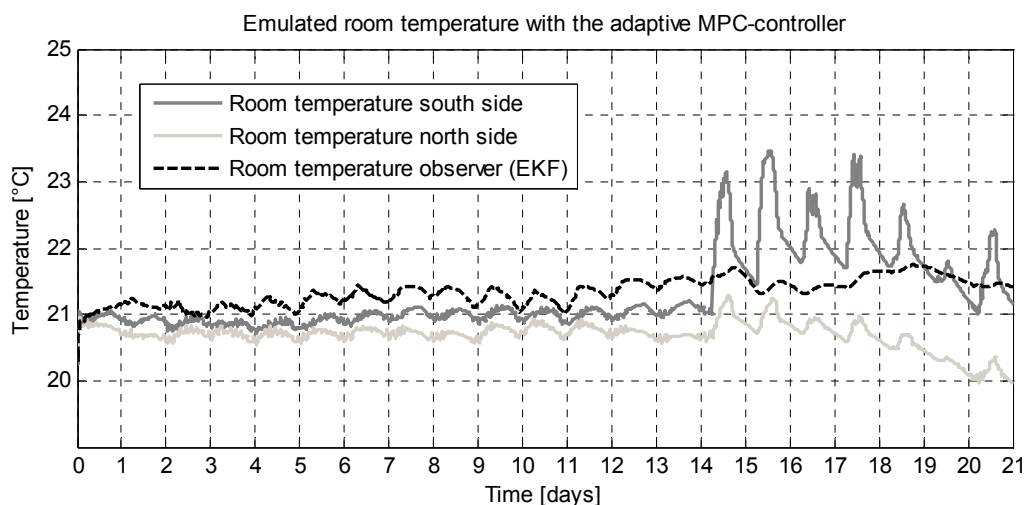


The MRML identification algorithm has been combined with the MPC controller and the resulting adaptive MPC controller has been compared with conventional heat pump controllers. The controllers have been tested in a test bench with the capability of house and earth probe emulation (Bianchi 2005, Bianchi 2007). The results show that the adaptive MPC controller was able to estimate the correct parameters in a few days and to optimize the house heating.



**Figure 9: Results of the adaptive MPC controller with the MRML identification algorithm**

The algorithm was also able to detect and compensate the mean solar gain as a flattening of the heating curve (third week of the emulation, see Figure 10). Due to the unmodeled solar effects on the room temperature, the identification of the time constant in the third week was more difficult. As shown in Figure 10, the EKF was able to estimate the mean course of the room temperature of the house.



**Figure 10: Emulated and estimated room temperatures (with an EKF)**

## 5 NOMENCLATURE

### 5.1 Symbols

Symbol	Unit	Description
$c_r$	$[J / (Kg \cdot K)]$	Mean specific capacity of the walls
$e$		White noise
$F(\theta), G(\theta), H(\theta)$		System matrices of the time-discrete state-space model
$K(\theta)$		Kalman gain
$V_r$	$[m^3]$	Mean volume of the house
$V_w$	$[m^3]$	Mean volume of the heating water
$T_{HW,In}$	$[K]$	Return flow temperature of the heating water
$T_{HW,Out}$	$[K]$	Flow temperature of the heating water
$T_R$	$[K]$	Room temperature
$T_A$	$[K]$	Outdoor temperature
$\dot{Q}_{HP}$	$[W]$	Heat pump heat flow
$\dot{Q}_{RA}$	$[W]$	Heat flow from the house to the ambient
$\dot{Q}_{WR}$	$[W]$	Heat flow from the heating water to the room
$\dot{Q}_{Stat}$	$[W]$	Mean heat flows from internal sources
$u$		Input signal vector
$V_N$		Loss function
$x$		State vector
$\hat{x}$		Estimation for $x$
$y$		Output signal vector
$\hat{y}$		Estimation for $y$
$\Phi$		Regressor matrix
$\hat{\theta}$		Estimation for $\theta$
$\varepsilon$		Residuals ( $y - \hat{y}$ )
$\kappa_{loss}$	$[W / K]$	Heating curve of the house
$\kappa_{WR}$	$[W / K]$	Mean specific conductivity of the floor
$\Lambda$		The covariance matrix of of $y$
$\Sigma$		Covariance matrix of the state vector error $x - \hat{x}$
$\rho_r$	$[Kg / m^3]$	Mean density of the walls
$\tau_{wall}$	$[s]$	Mean time constant of the walls

### 5.2 Acronyms

Acronym	Description
ARX	Controlled autoregressive model
EKF	Extended Kalman filter
KF	Kalman filter
LS	Least squares
MPC	Model predictive control
MRML	Modified recursive maximum likelihood method
OE	Output error
PEM	Prediction error method
PWM	Pulse width modulation
RLS	Recursive least squares
RPEM	Recursive prediction error method
RML	Recursive Maximum Likelihood method

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