

FIELD TESTS OF HEAT EXTRACTION OR INJECTION USING FOUNDATION PILES AS GROUND HEAT EXCHANGERS APPLYING LOW FLOW CIRCULATION

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Abstract: Field experiment of heat extraction or injection with foundation piles as ground heat exchangers applying low flow circulation has been carried out since October 2006 in order to develop the low flow ground heat exchanger. The ground heat exchanger can extract or inject nearly equal heat to the conventional ground heat exchanger even in a case where the low flow circulation system is adopted in the primary side. As the results of experiment, it was confirmed that several types of ground heat exchangers with flow rate of 8L/min extracted nearly equal heat to the conventional one with flow rate of 16L/min. The authors also conducted the experiment using foundation piles with the different specification. Additionally, temperature variation calculated by a simulation program is validated by comparing to the measured one.

Key Words: *Ground Source Heat Pump, Ground Heat Exchanger, Foundation Pile, Low Flow Circulation, Field Test*

1 INTRODUCTION

In the ground source heat pump (GSHP) system design, it is recommended that flow rate of thermal medium in the ground heat exchanger (GHEX) is kept enough large to prevent decrease of heat extraction or injection from the ground via the GHEX due to laminar flow. Previously, the authors have conducted heat extraction test with steel foundation piles as GHEX. They were the direct circulation type with the laminar flow of thermal medium and the indirect type with U-tubes and turbulent flow. As the result, the amount of heat extraction via the indirect type GHEX with double U-tube was almost equal to the one via the direct circulation type GHEX. The authors designed that double U-tubes was inserted to each piles basis on the result when the GSHP system utilizing steel foundation piles was installed to actual building. Then the flow rate was kept more than 8 L/min/U-tube in order to maintain turbulent flow. However, it is more difficult to maintain flow rate of the thermal medium enough large when the low flow circulation system is adopted in the primary side. In this paper, field test of heat extraction or injection using foundation piles as GHEX applying low flow circulation in order to develop the low-flow ground heat exchanger (LF-GHEX). The LF-GHEX can extract or inject nearly equal heat to the conventional GHEX even in the case where low flow circulation system is adopted in the primary side. First, the authors investigate decrease of heat extraction or injection by reduction of the flow rate. Next, outlines and results of field tests of the LF-GHEX were introduced. In addition, the temperature variation calculated by a simulation program for the GHEX, which is needed to predict performance of the GSHP system applying low flow circulation, is validated by comparison with the measured one.

2 ISSUE OF APPLYING LOW FLOW CIRCULATION SYSTEM TO GROUND HEAT EXCHANGER

2.1 Heat extraction or injection performance of ground heat exchanger

The most important factor that influences heat extraction or injection via the GHEX is temperature of the thermal medium in the GHEX. For example, heat extraction becomes larger when the temperature drops. This is due to increase of temperature difference between the thermal medium and the ground.

Here, if the undisturbed ground temperature is regarded as the representative ground temperature, the heat extraction rate of GHEX per the temperature difference is calculated by the following equation.

$$q'_p = q_p / (T_{pm} - T_{s0}) = q_p / \Delta T_{pm} \quad (1)$$

This means heat extraction or injection performance of any given GHEX.

On the other hand, assuming that heat extraction via the GHEX Q_p is constant, the temperature variation of thermal medium ΔT_{pm} can be expressed by the following equation.

$$\Delta T_{pm} = R_{p-out} Q_p / A_{p-out} - \Delta T_s(r_{p-out}, t) \quad (2)$$

This equation indicates that the internal and external thermal resistance of the GHEX shown in Figure 1 affects the temperature variation of thermal medium. The first term in the right side of Equation (2) varies by the internal thermal resistance, which is the one from thermal medium circulated in the GHEX to surface of the GHEX. It can be treated as constant value if a certain level of the time elapses. The second term depends on the external thermal resistance. It points effective thermal conductivity of the ground surrounding the GHEX, which includes not only pure thermal conductivity but also ground water advection. When the GHEX in the ground is regarded as an infinite follow cylinder in the infinite isotropic solid, it is possible to calculate the temperature variation $\Delta T_s(r_{p-out}, t)$ by applying the cylindrical heat source theory.

$$\Delta T_s(r_{p-out}, t) = - \frac{2Q_p}{A_{p-out} \pi \lambda_s} \int_0^\infty (1 - e^{-a_s u^2 t}) \frac{J_0(ur) Y_1(ur_{p-out}) - Y_0(ur) J_1(ur_{p-out})}{u^2 [J_1^2(ur_{p-out}) + Y_1^2(ur_{p-out})]} du \quad (3)$$

This equation suggests that $\Delta T_s(r_{p-out}, t)$ varies with the time elapse if the heat in the ground is transferred by only conduction. Therefore, q'_p and ΔT_{pm} also depend on the elapsed time.

2.2 Issue of applying low flow circulation system

In design process of the GSHP system, it is recommended that flow rate of thermal medium circulated in the GHEX is kept enough large to maintain the turbulent flow. This is because laminar flow of the thermal medium yields reduction of convective heat transfer in the GHEX. For example, Reynolds number and heat convection rate according to flow rate per U-tube with the inside diameter of 25 mm are shown in Figure 2. The Reynolds number Re and heat convection rate α are calculated by the following equations.

$$Re = \frac{ud}{\nu} \quad (4)$$

$$\alpha = \frac{Nu \lambda}{d} \quad (5)$$

Here,

$$\begin{aligned}
 N_u &= 3.65 & (R_e < 2100) \\
 N_u &= \frac{(C_f/2)(R_e - 1000)P_r}{1 + 12.7\sqrt{C_f/2}(P_r^{2/3} - 1)} \\
 C_f &= 0.079/R_e^{0.25} & (R_e > 3000)
 \end{aligned} \tag{6}$$

Especially in the case of heat extraction with $T_{pout} = 2\text{ }^{\circ}\text{C}$, the Reynolds number becomes less than 2100 with the flow rate of around 6.5 L/min. Then, it is predicted that the heat extraction performance of GHEX reduces due to the decrease of heat convection. Therefore in order to prevent performance drop of the GSHP system, it is more important to decide specification of the LF-GHEX, which can extract or inject nearly equal heat to the conventional GHEX.

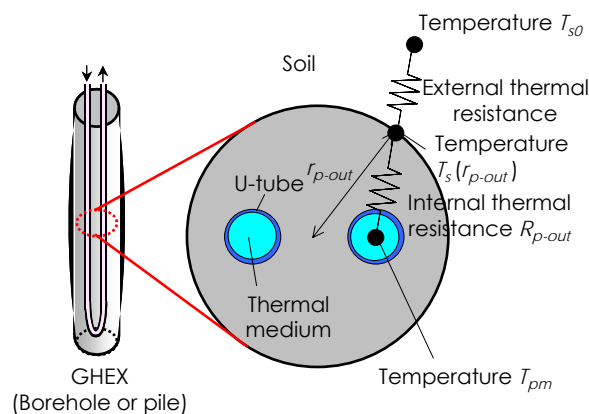


Figure 1: Concept diagram of internal and external thermal resistance of GHEX

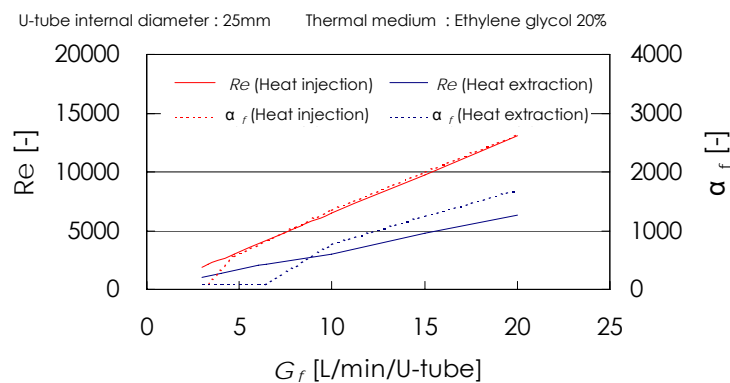


Figure 2: Reynolds number and heat convection rate according to flow rate

3 FIELD TEST OF HEAT EXTRACTION OR INJECTION USING FOUNDATION PILES AS GROUND HEAT EXCHANGERS APPLYING LOW FLOW CIRCULATION

3.1 Outlines of field experiment

3.1.1 Experimental set-up

Field experiment of heat extraction or injection using foundation piles as GHEX applying low flow circulation has been conducted since November 2006.

Shown in Figure 3 is plane view at the field test site in which arrangement of foundation piles as GHEX and their specification are indicated. There are 11 piles in total in this experimental site. The eight piles are connected to the control equipment for heat extraction or injection test as shown in Figure 3. The one is a pre-casting concrete pile and the others are steel foundation piles. The three piles with small diameter of $\Phi 267$ mm are buried with small interval of 0.9 m each other. We call these piles 'Group-pile'. The other three piles, which have diameter of $\Phi 400$ mm, $\Phi 800$ mm, and $\Phi 1200$ mm respectively, are connected to the thermal response test apparatus.

Figure 4 draws schematic diagram of the experimental set-up for the heat extraction or injection test. Each pile is used as GHEX by inserting U-tubes and filling water. The effective length as GHEX is 25 m. The control equipment can provide water with constant temperature and constant flow rate. The water can be supplied to up to two piles at the same time. Figure 5 shows schematic diagram of thermal response test using steel foundation piles with large diameter. The steel foundation pile as GHEX is connected to the thermal response test apparatus, which can supply water with constant heating and constant flow rate.

3.1.2 Foundation piles as ground heat exchanger

Examples of foundation piles as GHEX are shown in Figure 6. Two types of U-tube are used for this experiment. The first one is made of HDPE and the internal diameter is 26 mm (ISO standard of 25 A). This one is also used for general borehole GHEX. The other one is made by shaping the commonly used XLPE pipe with the internal diameter of 16 mm (JIS standard of 16 A) like the U-tube. The former is little more expensive than the latter, but it makes the construction works more easily. The reason is that it is inserted into the piles with large diameter more easily due to the hardness.

3.1.3 Measurement point

Measurement points are indicated in Figure 4 and Figure 5. In the heat extraction or injection test, Pt-100 sensors as temperature measurement points are set at inlet and outlet of each GHEX. Water temperature in each GHEX is measured by T-type thermo couples at 6.5, 13.5, 20.5, 27.2 m deep. Internal surface temperature of the pile and external surface temperature of U-tube are also observed at 13.5 m deep for each pile. Electromagnetic flow meters are equipped in each line that leads to the GHEX. Pressure differences between inlet and outlet of are measured by the differential manometers.

In the thermal response test, Pt-100 sensors are placed at inlet and outlet of the GHEX. An electromagnetic flow meter is installed in the thermal response test apparatus. Three points in the GHEX are also measured by the thermo couples.

3.1.4 Test procedure and experimental conditions

Shown in Figure 7 is an example of heat injection test in which inlet and outlet temperature of the GHEX and flow rate are drawn. The inlet temperature is kept at 2 °C constant for every heat extraction test and 25 °C constant for every heat injection test, respectively. The flow rate is changed only at the moment of changing the condition and kept constant during the experiment. As shown in Figure 7, this experiment is conducted for 170 h (1 week)~340 h (2 weeks) per one experimental condition. After the time elapses 250 h for Exp.1 and 100 h for Exp.2 ~Exp. 4, the outlet temperature is almost stable and it is possible to consider the temperature variation achieves at semi-steady state. Thus, the authors changed the experimental condition after the achievement had been confirmed.

Table 1 indicates the experimental conditions that are the pile used for the experiment, specification of the GHEX, flow rate per the GHEX, and the inlet temperature. This test was conducted in order to compare the heat extraction or injection performance in the case where the following conditions differ.

Case1 GHEX with HDPE U-tubes of 25 A and XLPE U-tubes of 16 A

Case2 Steel foundation piles and pre-casting concrete pile

Case3 Specification of steel foundation piles such as diameter and Group-pile

3.1.5 Performance evaluation of ground heat exchangers

In this paper, heat extraction rate q'_p describe above is used for performance evaluation of the GHEX. By using the measured value, the internal thermal resistance of GHEX R_{p-out} can be calculated by the following equation.

$$R_{p-out} = -A_{p-out} (T_f - T_s(r_{p-out}, t)) / Q_p \quad (7)$$

Here, the heat extraction Q_p is expressed by the following equation.

$$Q_p = c_f \rho_f G_f (T_{pout} - T_{pin}) \quad (8)$$

The average values of temperatures and flow rate for the last 2days' experiment are used for this calculation. The thermal resistance R_{p-out} calculated by Equation (7) is substituted to Equation (2). Also, if the elapsed time $t = 1000$ h, effective thermal conductivity $\lambda_s = 1.5$ W/m/K, and thermal diffusivity $a = 5.0 \times 10^{-7}$ m²/s are given to Equation (2), the temperature variation $\Delta T_s(r_{p-out}, t)$ can be calculated by Equation (3). As the result, we can obtain the temperature variation ΔT_{pm} and the heat extraction rate q'_p by using Equation (2) and Equation (1), respectively.

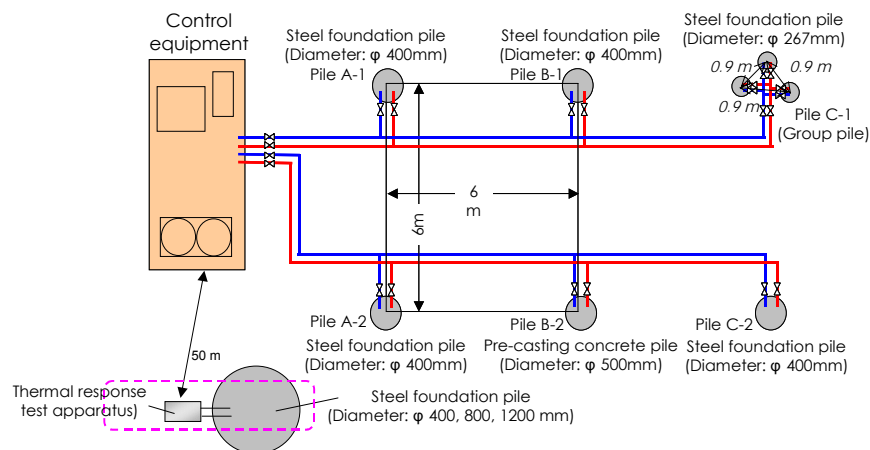


Figure 3: Plane view at the field test site

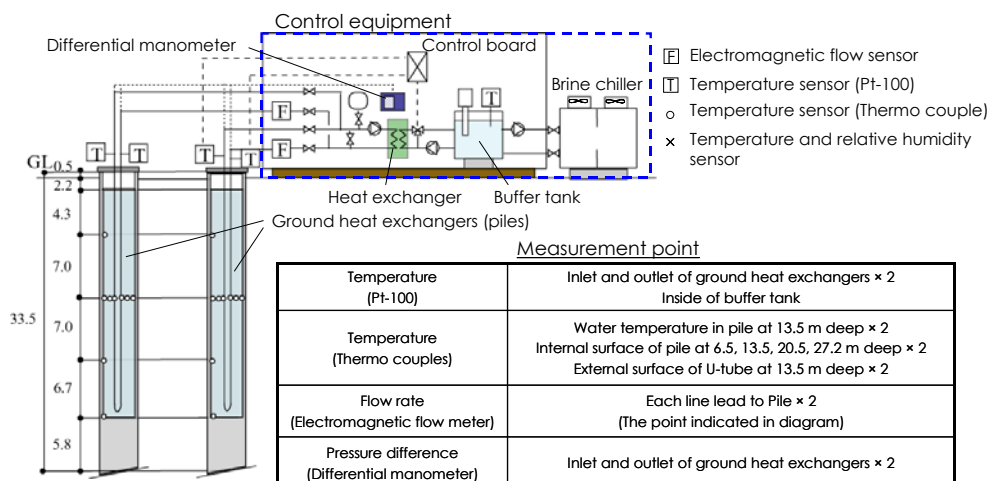
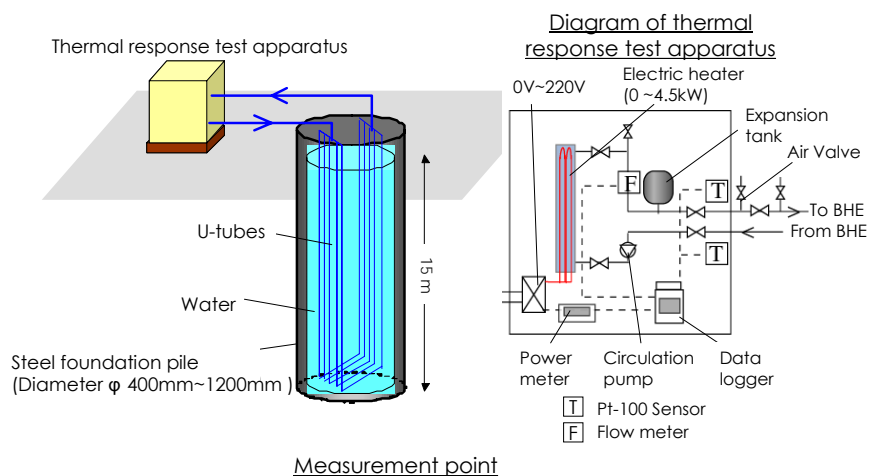


Figure 4: Schematic diagram of experimental set-up for heat extraction or injection test



Measurement point

Temperature (Pt-100)	Inlet and outlet of ground heat exchangers
Temperature (Thermo couples)	Water temperature in pile at 8 m deep Internal surface of pile at 8 m deep External surface of U-tube at 8 m deep
Flow rate (Electromagnetic flow meter)	Inside of the thermal response test apparatus

Figure 5: Schematic diagram of thermal response test with foundation pile

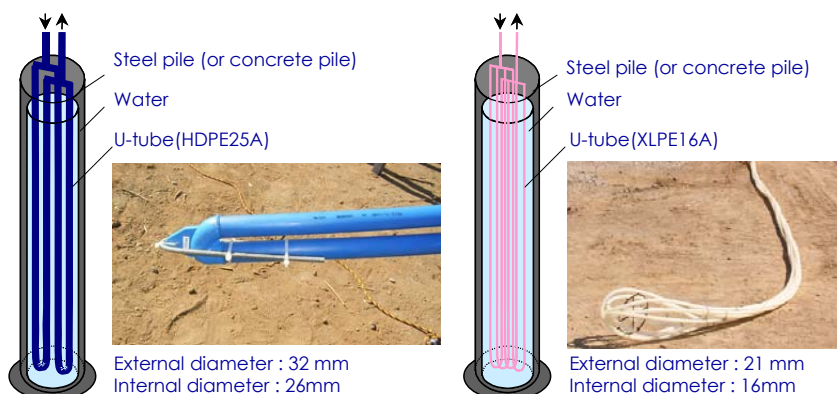


Figure 6: Foundation piles as GHEX

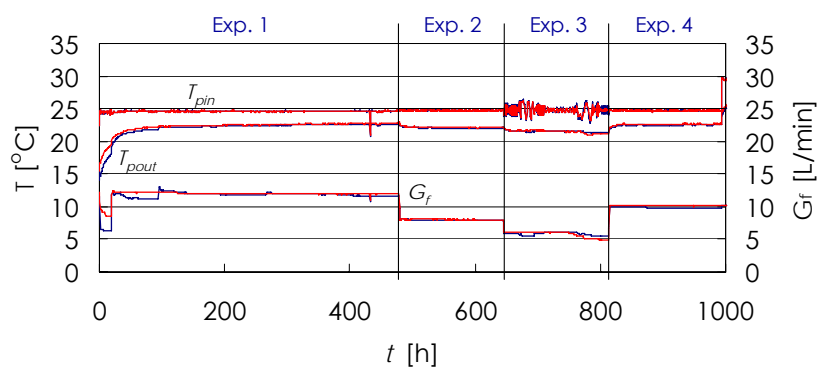


Figure 7: Example of heat injection test

Table 1: Experimental conditions

		Pile	Specification	G_f [L/min]	T_{pm} [°C]
CASE1	1-1	Pile A-1 (φ 400mm)	16A U-tube × 3	6,8,10,12	25
	1-2		16A U-tube × 4		
	1-3	Pile A-2(φ 400mm)	16A U-tube × 5		
	1-4	Pile B-1 (φ 400mm)	25A U-tube × 2	6,8,10,12,16	2
	1-5		16A U-tube × 5		
CASE2	2-1	Pile C-1 (φ 267mm)	25A U-tube × 1	4,8,12	2
	2-2	Pile C-2 (φ 400mm)		8,16,24	
	2-3	Pile B-2 (Concrete pile)		8,16	
CASE3	3-1	Steel pile with φ 400mm	25A U-tube × 2	30	TRT
	3-2	Steel pile with φ 800mm	25A U-tube × 4	8,16,26	TRT
	3-3	Pile C-1 (φ 267mm × 2)	25A U-tube × 1	8,16,24	2
	3-4	Pile C-1 (φ 267mm × 3)	25A U-tube × 1		2

3.2 Result and discussions

3.2.1 Performance of ground heat exchangers

Figure 8 shows heat extraction rate q'_p according to flow rate per the GHEX G_f in the heat injection test of CASE1. Heat extraction rate of the conventional GHEX with two HDPE 25 A U-tubes and $G_f = 16$ L/min is also shown in Figure 8. When the number of XLPE 16 A U-tubes is four or five, the heat injection rate of GHEX with $G_f = 8$ L/min is almost equal to that of the conventional GHEX. The heat injection rate of GHEX with $G_f = 12$ L/min is similar to that of the conventional GHEX in the case where the GHEX with three XLPE 16 A U-tubes are used. These results indicate that increasing surface area due to utilizing more number of U-tubes raises heat injection rate but the effect becomes smaller as increase of the number.

Additionally, heat extraction rate q'_p according to flow rate per the GHEX G_f in the heat extraction test of CASE1 is shown in Figure 9. The heat extraction rate of GHEX with five XLPE 16 A U-tubes and $G_f = 8$ L/min is nearly equal to the one of the conventional GHEX as well as in the heat injection test.

In Figure 10, heat extraction rate q'_p according to flow rate G_f in the heat extraction test of CASE2 is compared. Although the pre-casting concrete pile has larger diameter than the steel foundation pile with external diameter of φ400 mm, the heat extraction rate of concrete pile is smaller than the one of steel foundation pile. The heat extraction rate of concrete pile with internal diameter of 300mm is close to the one of steel foundation pile with external diameter of φ267 mm and internal diameter of 255 mm. The reason is that the totals of internal and external thermal resistance of these piles are almost the same due to two reasons. The one is that both of the concrete thermal conductivity and the soil effective thermal conductivity are around 1.5 W/m/K. In addition, the heat convections of filled water, whose intensity depends on the internal diameter, are the similar to each other.

Shown in Figure 11 is heat extraction rate q'_p according to flow rate G_f in the heat extraction or injection test of CASE3. The heat extraction rate of steel foundation pile with external diameter of φ800 mm is approximately 1.5 times larger than the one of steel foundation pile with external diameter of φ400 mm. With regard to the Group-pile, when the number of steel foundation piles adds up from two to three, the heat extraction rate also becomes about 1.5 times.

3.2.2 Pressure drop of ground heat exchangers

Figure 12 compares pressure drops of ground heat exchangers used in CASE1. The pressure drops of the GHEX with four U-tubes, $G_f = 8$ L/min and five U-tubes, $G_f = 10$ L/min,

whose flow rates per U-tube are the same, are close to each other. Additionally, the pressure drop decreases as the reduction of flow rate. This result indicates that reduction of flow rate also makes electric power of the circulate pump lower due to the decrease of pressure drop.

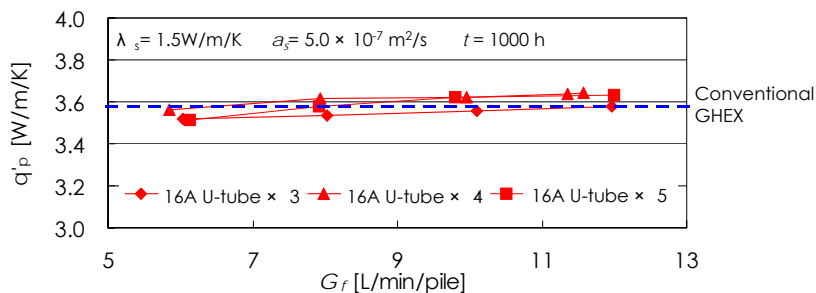


Figure 8: Heat extraction rate according to flow rate in heat injection test of CASE1

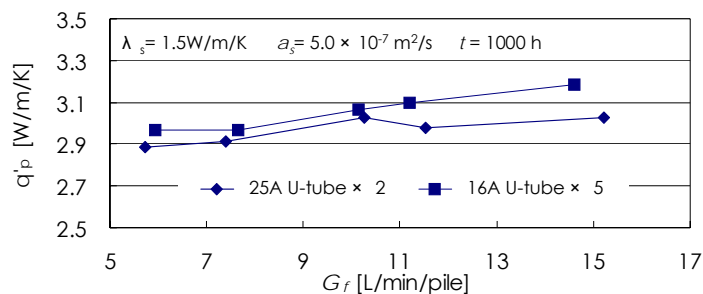


Figure 9: Heat extraction rate according to flow rate in heat extraction test of CASE1

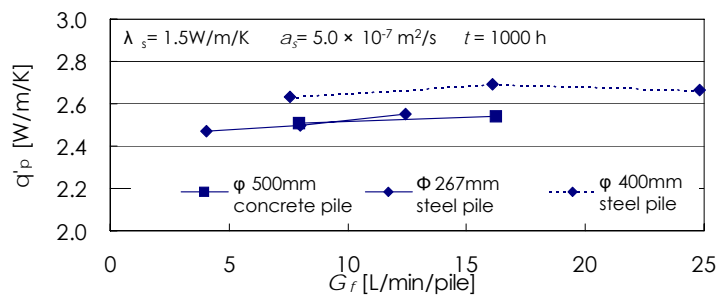


Figure 10: Heat extraction rate according to flow rate in CASE2

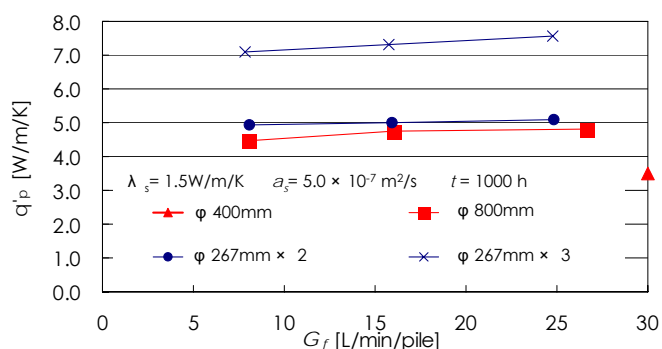


Figure 11: Heat extraction rate according to flow rate in CASE3

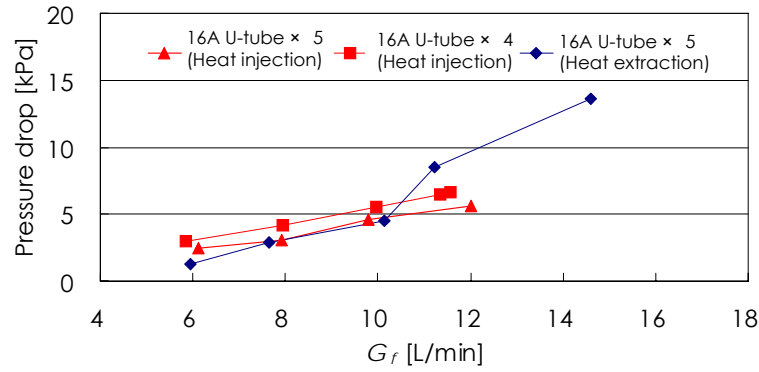


Figure 12: Comparison of pressure drops in CASE 1

4 VALIDATION OF CALCULATION RESULT BY SIMULATION PROGRAM FOR THE GROUND HEAT EXCHANGER

4.1 Outlines of simulation program

4.1.1 Heat transfer equation in the ground heat exchanger

The fundamental algorithm of this simulation program is similar to the one of the design tool for the GSHP system that the authors developed. However, this one is modified to calculate temperatures of each part in the GHEX with large diameter like foundation piles more accurately. Figure 13 indicates concept diagram of the simulation program. The heat balance equations of each part are described the following.

When the thermal medium in U-tubes is regarded as one mass system, the following differential equation is given.

$$c_f \rho_f V_f \frac{dT_f}{dt} = -c_f \rho_f G_f (T_{pin} - T_{pout}) + K_{u-out} A_{u-out} (T_w - T_f) \quad (9)$$

Also, the following equation is obtained by regarding the filled water as one mass system as well as the thermal medium.

$$c_w \rho_w V_w \frac{dT_w}{dt} = K_{u-out} A_{u-out} (T_f - T_w) + K_{p-out} A_{p-out} (T_s(r_{p-out}, t) - T_w) \quad (10)$$

Overall heat transfers K_{u-out} and K_{p-out} are expressed by the followings.

$$K_{u-out} = \frac{1}{r_{u-out} \left(\frac{1}{r_{u-out} \alpha_{uw}} + \frac{1}{\lambda_u} \ln \frac{r_{u-out}}{r_{u-in}} + \frac{1}{r_{u-in} \alpha_f} \right)} \quad (11)$$

$$K_{p-out} = \frac{1}{r_{p-out} \left(\frac{1}{r_{p-in} \alpha_{pw}} + \frac{1}{\lambda_p} \ln \frac{r_{p-out}}{r_{p-in}} \right)} \quad (12)$$

Here, heat convection rate α_{uw} and α_{pw} are calculated by Equation (5). Then, the Nusselt numbers can be calculated by the following equations if the GHEX has large diameter.

$$N_u = 0.13R_a^{1/3} \quad (13)$$

Here,

$$R_a = G_f P_r \quad (14)$$

Additionally, the Grashof number is given as the following

$$G_r = \frac{l^3 g_w \beta_w \Delta T}{\nu_w^2} \quad (15)$$

When the heat convection rate α_{uw} is calculated, the length of U-tube l_U and temperature difference $\Delta T = T_w - T_{u-out}$ are used. Also, the length of pile l_p and temperature difference $\Delta T = T_{p-in} - T_w$ are given for calculate α_{pw} .

4.1.2 Calculation flow

Figure 14 shows the calculation flow. This simulation program calculates T_{pou} by giving hourly T_{pin} and G_f as the calculated conditions. The temperatures T_{u-out} and T_{p-in} are calculated by the iteration method.

The temperature T_{u-out} is obtained by comparing the calculation results by the following equations.

$$\begin{aligned} Q_{fw} &= K_{u-out} A_{u-out} (T_w - T_f) \\ &= \frac{1}{\left\{ r_{u-out} \left(\frac{1}{\lambda_u} \ln \frac{r_{u-out}}{r_{u-in}} + \frac{1}{r_{u-in} \alpha_f} \right) \right\}} A_{u-out} (T_{u-out} - T_f) \end{aligned} \quad (16)\text{-A}$$

$$= \alpha_w A_{u-out} (T_w - T_{u-out}) \quad (16)\text{-B}$$

Also, comparison of Q_{pw} calculated by Equation (17)-A and Equation (17)-B provides the temperature T_{p-in} .

$$\begin{aligned} Q_{pw} &= K_{p-out} A_{p-out} (T_s(r_{p-out}, t) - T_w) \\ &= \frac{1}{\left\{ r_{p-out} \left(\frac{1}{\lambda_p} \ln \frac{r_{p-out}}{r_{p-in}} \right) \right\}} A_{p-out} (T_s(r_{p-out}, t) - T_{p-in}) \end{aligned} \quad (17)\text{-A}$$

$$= \alpha_w A_{p-out} (T_{p-in} - T_w) \quad (17)\text{-B}$$

4.2 Validation of calculation result

In order to validate the calculation result, the temperature T_{pout} calculated by the simulation program is compared to the experimental result. As the calculated conditions, the ground temperature is set at 11.0 °C, which is the undisturbed ground temperature in this site. Typical soil heat capacity of 3000 kJ/m³ is used and effective thermal conductivity of 1.5 W/m/K is given from the result of the thermal response test conducted before this experiment. Figure 15 compares variations of the measured and calculated T_{pou} . Variations of the measured T_{pin} and G_f , which are given as the calculated conditions, are also shown in Figure 15. Difference of temperature variation between measurement and calculation is hardly observed. Since the calculated temperature variation produces good agreement with the measured one, this simulation program can be used for performance prediction of the GSHP system applying low flow circulation.

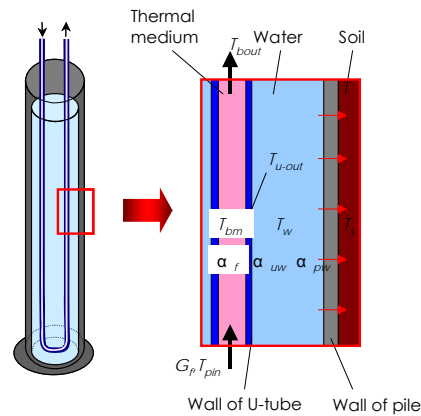


Figure 13: Concept diagram of simulation program for GHEX

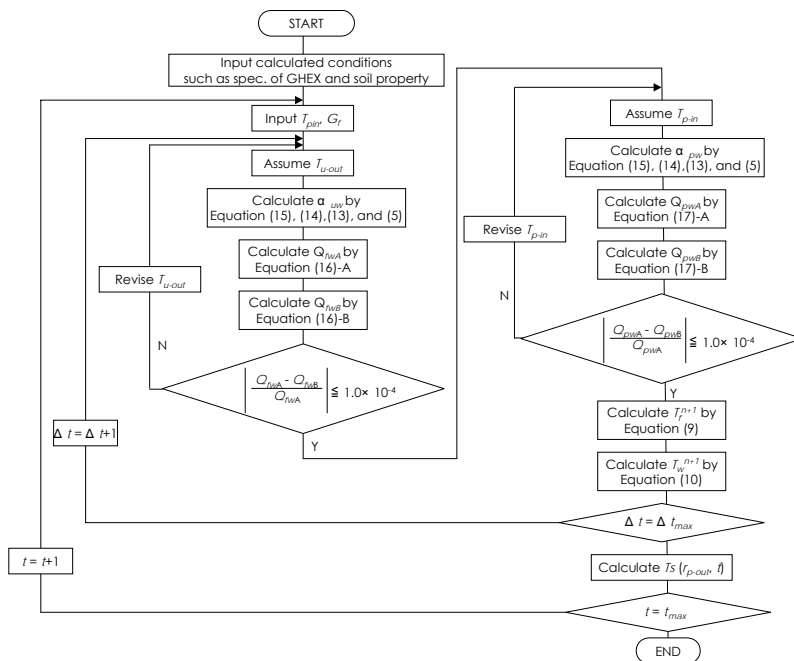


Figure 14: Calculation flow of simulation program for GHEX

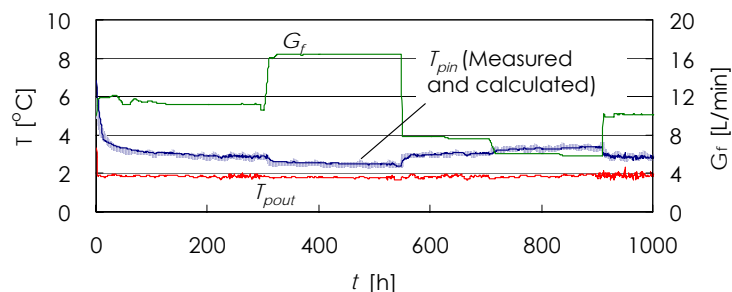


Figure 15: Comparison between temperature variations of measurement and calculation

5 SUMMARIES

- 1) As the results of heat extraction or injection using foundation piles as ground heat exchangers, several types of GHEX with flow rate of 8L/min extracted nearly equal heat to the conventional one with flow rate of 16L/min.

- 2) The heat extraction rate of concrete pile with external diameter of $\Phi 500$ mm and internal diameter of 300mm was similar to the one of steel foundation pile with external diameter of $\Phi 267$ mm and internal diameter of 255 mm.
- 3) The heat extraction rate of steel foundation pile with external diameter of $\Phi 800$ mm was approximately as 1.5 times as the one of steel foundation pile with external diameter of $\Phi 400$ mm. When the number of steel foundation piles was from two to three, the heat extraction rate was also about 1.5 times.
- 4) Since the temperature variation calculated by the simulation program for the GHEX produced good agreement with the measured one, this program could be used for performance prediction of the GSHP system applying low flow circulation.

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NOMENCRATURE

A : Area [m^2], a : Thermal diffusivity [m^2/s], c : Specific thermal capacity [$kJ/kg/K$], d : Diameter [m], G : Flow late [m^3/s], g : Gravitational constant [m^2/s], Gr : Grashof number [-], J_x : xth-order Bessel function of first kind, K : Overall heat transfer coefficient [$W/m^2/K$], l : Length [m], Nu : Nusselt number [-], Pr : Prandtl number [-], Q : Heat extraction rate [W], q : Heat extraction rate per length [W/m], q' : Heat extraction rate per length and temperature difference [$W/m/K$], R : Thermal resistance [m^2K/W], r : Radius [m], Ra : Rayleigh number [-], Re : Reynolds number [-], T : Temperature [$^{\circ}C$], t : Time [h], u : Characteristic value [h], V : Volume [m^3], v : Velocity [m/s], Y_x : xth-order Bessel function of second kind, α : Heat convection rate [$W/m^2/K$], β : Coefficient of cubic expansion [K^{-1}], λ : Thermal conductivity [$W/m/K$], ρ : Density [kg/m^3]

Subscript

f : Thermal medium, p : GHEX (Pile), s : Soil, u : U-tube, w : water, m : mean, in : Inlet, out : Outlet, $-in$: Inside, $-out$: Outside